Ottimizzazione delle osservazioni di sorgenti transienti con l’osservatorio gamma CTA

Optimization of transient sources observation for the gamma-ray observatory CTA

Candidato: Matteo Giomi
Relatore: Dr. Gernot Maier
Corelatore: Prof. Oscar Adriani

Anno Accademico 2012/13
Acknowledgements

I would like to thank all the people in DESY Zeuthen for the kind welcome and hospitality I have received. In particular I would like to thank Gernot Maier for his patience and precious guidance during my year in Zeuthen. I am very grateful to Lucie Gerard for the countless times she has helped me in this work. Nathan Kelley-Hoskins is also to be acknowledged for his availability, kindness, and stimulating questions. I must thank Alessio Tiberio and the whole Ungulate Crew for helping me fighting the bureaucracy from the distance. A special thanks goes to Oscar Adriani for the courtesy he has always shown, both as a co-supervisor and as a professor.
## Contents

**Introduction** 6

1 Imaging Atmospheric Cherenkov Technique 9
   1.1 Extensive air showers ................................. 10
   1.1.1 Cherenkov Emission of Air Showers ............... 13
   1.2 Cherenkov telescopes design .......................... 15
   1.3 Data analysis ........................................ 17
   1.4 Current Status of IACT ............................... 21
   1.5 Future development, The Cherenkov Telescope Array ... 24
   1.5.1 CTA design concept ................................ 26
   1.5.2 Array layouts and performances ................. 28

2 Simulation of VHE light curves from Active Galactic Nuclei 33
   2.1 Active Galactic Nuclei ............................... 34
   2.2 Blazars .............................................. 35
   2.3 Simulation of AGN light curves ....................... 38
   2.3.1 Characterizing VHE AGN variability .............. 39
   2.3.2 Light curve simulation algorithm ................ 43
   2.3.3 Simulated light curves parameter space .......... 50

3 Observing variable sources with CTA 51
   3.1 Implementation of the observing strategies ........ 52
   3.1.1 Pointing schemes .................................. 53
   3.1.2 Instrument response functions .................... 54
   3.2 Quantifying observing strategies performances .... 57
   3.2.1 Simulation loop ................................... 57
   3.2.2 Probability of detection .......................... 60
   3.2.3 Observing strategies parameter space .......... 61

4 Selection of the best observing strategies 65
   4.1 Observing strategies performances .................. 66
4.1.1 Detection probability in the $(T_{\text{win}}, T_{\text{obs}})$ plane \ldots \ldots 66
4.1.2 Detection probability as a function of the observing time 68
4.2 Best strategy to detect a source in as little time as possible \ldots 71
4.3 Best AGN monitoring strategy \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 74
4.4 Impact of light curves’ characteristics \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 77
  4.4.1 Impact of light curves’ characteristics on the detection probability \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 77
  4.4.2 Impact of light curves’ characteristics on the best strategy for fast detection \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 80
  4.4.3 Impact of light curves’ characteristics on the best monitoring strategies \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 81
4.5 Application to a current IACT: Monitoring of Mrk 421 with VERITAS \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 83

Outlook and conclusions 85
Introduction

The Cherenkov Telescope Array (CTA) is the new ground-based very high energy (VHE; \( E \gtrsim 100 \text{ GeV} \)) \( \gamma \)-ray observatory whose first light is planned for 2016. CTA is a world-wide project, building on the proven technology of Imaging Atmospheric Cherenkov Telescopes (IACTs). IACTs allow us to measure the energy and direction of incoming VHE \( \gamma \)-rays by imaging the Cherenkov light emitted during the development of the \( \gamma \)-ray-initiated atmospheric shower. Through the deployment of around 50-100 of such telescopes of three different sizes, CTA will be ten times more sensitive than current IACTs, significantly extending our current knowledge in high-energy astrophysics.

Roughly one third of the sources detected by IACTs are blazars. Blazars are a particular class of active galactic nuclei (AGN), supermassive black holes accreting matter at the center of some galaxies. Blazars are highly variable sources, on time scales from minutes to years. Their fluxes change by up to orders of magnitudes.

With a small field of view and a duty cycle reduced to \( \sim 1000 \) hours per year, IACTs require careful planning of the observations. AGN variability makes such planning a challenge. In this work we develop a tool to optimize the strategies for the observation of variable sources with CTA. CTA observing strategies are modeled taking into account the number of telescopes participating in the observation as well as the length of the observing windows and their repartition in time. The fluxes of blazars and their variations in time are simulated and used to characterize the observing strategies’ performances.

This thesis is divided into four chapters. The first introduces IACTs, their operation and CTA. The second present AGN and their variability at VHE, as well as the way the variability is simulated. The third chapter is dedicated to the developed toy model of the observing strategies and the procedure used to quantify their performance. Finally, the fourth chapter describes the optimization of CTA strategies for two different observing goals: detection of the source in the minimum amount of time and source monitoring. The
impact of AGN variability properties on observing strategies optimization is also presented.
Chapter 1

Imaging Atmospheric Cherenkov Technique

Gamma-rays flux from cosmic sources exhibit power law spectra; their flux decreases roughly by three order of magnitude for every decade in energy. Different detection techniques must be adopted depending on the energy range that has to be explored. While observations up to $\sim 100$ GeV can be successfully carried out with spaceborne experiments such as, the Fermi Gamma-ray Space Telescope [1], ground based instruments are necessary to extend the observations towards higher energies. In the very high energy (VHE) range, from $\sim 100$ GeV to a few tens of TeV, $\gamma$-ray fluxes are of the order of $\sim 10^{-11}\text{cm}^{-2}\text{s}^{-1}$ for strong sources. Collection areas of the order of $10^4\text{ m}^2$ are needed to detect several photons within one hour of observation [2, 3], much larger than the typical square meter collection areas achievable on a satellite.

With ground based instruments energy and arrival direction of incoming $\gamma$-rays must be measured in an indirect way. Earth’s atmosphere is not transparent to VHE photons: upon interaction of the primary energetic $\gamma$-ray with air nuclei in the upper atmosphere a cascade of relativistic particles (EAS, extensive air shower) is produced. Ground based instruments for VHE gamma astronomy detect secondary products resulting from the development of the $\gamma$-ray initiated showers. Currently two different techniques are used for this purpose: Imaging Atmospheric Cherenkov Telescopes (IACTs) and direct detection of shower particles on the ground (shower sampling). These are complementary techniques: the energy range of a typical Cherenkov telescope system is $\sim [0.05, 50]$ TeV to be compared with $\sim [0.1, 100]$ TeV of a shower sampling experiment such as HAWC [4].

IACTs can reconstruct the energy and direction of the primary gamma from images of the air shower initiated by the VHE $\gamma$-ray. Shower images
are obtained focusing the faint and short pulse of Cherenkov light emitted by the ultra relativistic (UR) shower particles onto a pixelled camera. The main challenge of any ground based instrument for VHE $\gamma$-ray astronomy is represented by the huge background of charged cosmic rays (CR). IACTs exploit the differences between gamma- and nuclei-induced shower images to achieve an effective suppression of the CR background. The primary limitations of IACTs are small field of view and low duty cycle. Cherenkov telescopes can operate only on clear, almost moonless nights and in good weather conditions. As a consequence, only $\sim$1000 h per year are available for observations. The field of view of Cherenkov telescopes is limited to some square degrees by technical and budget constraint. Despite these limitations, IACTs possess superior sensitivity, a lower energy threshold and a better angular resolution compared to shower sampling experiments. These qualities have made IACTs the most powerful tools for $\gamma$-ray astronomy in the VHE domain [2].

In this chapter an overall introduction of IACTs will be given. In Section 1.1 the formation of extensive air showers will briefly be discussed, outlining the differences between gamma- and cosmic-ray-induced showers. The Cherenkov emission of air showers and the transmission of this light through the atmosphere will also be discussed. Section 1.2 will be devoted to present the principal features of an IACT system. The principles of event reconstruction and background rejection will be outlined in 1.3. A brief description the three most sensitive IACT systems currently operating will be given in Section 1.4. Finally, in Section 1.5 CTA will be described.

## 1.1 Extensive air showers

An extensive air shower is a cascade relativistic particles traveling through Earth’s atmosphere. It results from the interaction of an energetic cosmic particle with the atomic nuclei of air molecules in the higher layers of the atmosphere. As a result of this interaction, a number of secondary particles are produced. Given its energy is sufficient each secondary particle can in turn produce other particles, leading to an exponential growth of the number of particle participating in the cascade. This multiplicative process continues until the energy of the particles falls below the threshold for further particle production. For a VHE primary particle the cascade at its maximum development is composed of $10^3 - 10^6$ particles and can travel distances of several kilometers.

The development of an air shower is strongly influenced by the kind of interactions involved in the creation of new particles. When the shower is
initiated by a photon or an electron, the electromagnetic force will dominate and the resulting shower is said to be electromagnetic. If the primary is a nucleus, interaction via strong and weak forces will also occur and we speak of hadronic shower.

**Electromagnetic Showers**

For energies of interest in the formation of a γ-ray EAS the dominant processes of interaction with matter are pair production for photons and bremsstrahlung for electrons and positrons\(^1\) [5]. The characteristic length scale for bremsstrahlung is the radiation length \(X_0\), defined as the mean distance over which an electron loses \((1 - 1/e)\) of its initial energy, \(X_0 \simeq 37\) g cm\(^{-2}\) in air. For pair production the corresponding quantity is the mean free path and in air is approximately \(9/7 \cdot X_0 \simeq 48\) g cm\(^{-2}\).

The main features of an electromagnetic shower can be derived using the toy model introduced by Heitler [6]. In this model only bremsstrahlung and pair production are considered and the interaction length for both of these processes is assumed to be equal to \(X_0\). According to this model after each interaction length all the particles in the shower will undergo interaction, resulting in the production of two secondary particles, see Figure 1.1. At each interaction the energy is shared equally among the secondary particles produced.

The number of shower particles will increase exponentially with the atmospheric depth traveled by the shower, while average particle energy will decrease accordingly. The shower will continue to grow until the ionization losses becomes dominant over bremsstrahlung, i.e when the mean particle energy drops below the so called critical energy, \(E_C\), which is air is \(\approx 85\) MeV. When the mean particle energy is around the critical energy the shower reaches its maximum development; for energies below \(E_C\) it will cease to grow as no new particles are produced. The atmospheric depth at which the shower reaches its maximum development will increase logarithmically with the energy of the primary. The number of particles at the shower maximum is proportional to this energy. From this model it follows that the total path length of all the charged particle in the shower is proportional to the energy of the primary [8]. This relation is the basis of the energy measurements with Cherenkov telescopes since it allows to relate the energy of the primary to the total amount of Cherenkov light produced in the shower, being the number of Cherenkov photons emitted by an UR particle proportional to the distance it travels. Despite the simplicity of this model the predicted relations among shower characteristic and primary particle energy agree with results of more

\(^1\)For now on the word electrons will refer to both electrons and positrons.
realistic and sophisticated Monte Carlo (MC) models.

**Hadronic Showers**

Energetic Cosmic Rays entering the atmosphere undergo inelastic scattering with air nuclei, producing secondary mesons and nucleons as well as excited nuclear fragments. These particles continue to multiply by subsequent nuclear collisions until their energy drops below the threshold for multiple pion production, about 1 GeV. This cascade of heavy particles form the core of the shower. Beside this hadronic core, secondary electromagnetic showers are induced by neutral and charged mesons decays. At each interaction about one third of the energy is transferred from the hadronic core to the electromagnetic component. Eventually all the energy available for shower formation will appear in the electromagnetic component [5, 9].

There are profound differences among hadronic and electromagnetic showers: the interaction length for a 1 TeV proton in air is approximately 80 g cm⁻². Hadronic showers thus penetrate deeper in the atmosphere and the atmospheric depth of the shower maximum, \( X_{\text{max}} \), is on average larger. Secondary particles produced by strong and weak interactions have higher transverse momenta and longer interaction length compared to the electromagnetic case. As a result, hadronic shower have larger lateral extension than electromagnetic ones where the lateral spread of particles is mostly due to multiple
Coulomb scattering of electrons. Cherenkov radiating particles in a gamma-induced shower are, on average, closer to the direction of the primary. Finally, higher complexity in terms of number of particles involved in hadronic shower formation compared to the dominant three particle processes for the electromagnetic case leads to larger fluctuations in shower development and make them less regular in shape, as shown in Figure 1.2. These differences in shower morphology are exploited by IACTs to achieve a good suppression of the huge background of non-\(\gamma\)-ray events.

![Figure 1.2: Simulated vertical EAS originated by a 300 GeV \(\gamma\)-ray (left) and by a 1 TeV proton (right). The plot shows the secondary particles projected on a plane. The energy of the primary is different to take into account energy losses during hadronic shower formation.][3]

1.1.1 Cherenkov Emission of Air Showers

Given the energy involved in the formation of an EAS, shower particles move through the atmosphere at relativistic speed and can therefore be above the threshold for Cherenkov emission. The Cherenkov effect occurs when a charged particle traverses a dielectric medium at a speed \(v\) greater than the speed of light in that medium, i.e. when \(v \geq c/n\), \(n\) being the refractive index of the dielectric and \(c\) the speed of light in vacuum. The corresponding
energy threshold is then:

\[ E_{th} = \frac{mc^2}{\sqrt{1 - n^{-2}}} \]  

(1.1)

Since \( E_{th} \) is proportional to the particle’s mass, an air shower’s Cherenkov emission will be dominated by the contributions from electrons. Cherenkov radiation is emitted in a cone with aperture angle \( 2\theta_C \) with respect to the particle direction. The Cherenkov angle \( \theta_C \) is related to the particle velocity \( v = \beta c \) and to the refractive index of the medium \( n \):

\[ \cos \theta_C = \frac{1}{\beta n} \]  

(1.2)

Figure 1.3: (a) Geometry of Cherenkov emission for a single particle, from [10], (b) Cherenkov emission at different altitudes results in the formation of the Cherenkov light pool, from [7].

As the refractive index of air varies with height, both the Cherenkov threshold and the emission angle \( \theta_C \) will depend on the atmospheric altitude. With decreasing height the atmospheric density increases and so does the refractive index. For a particle with \( \beta \simeq 1 \) the Cherenkov angle at 10 km a.s.l is about 0.7° while it reaches 1.4° at sea level [3]. The Cherenkov energy threshold for electrons and positrons in air is \( \sim 20 \) MeV at sea level and \( \sim 40 \) MeV at 10 Km a.s.l. [11]. In this range of altitudes \( E_{th} \) is always well below the critical energy; all shower electrons will therefore emit Cherenkov radiation.
Cherenkov light has a continuous spectrum with most of the radiation emitted at short wavelengths. The number of Cherenkov photons with wavelength in \([\lambda, \lambda + d\lambda]\) emitted in a path length \(dx\) by a unitary charged particle is given by:

\[
\frac{d^2 N}{d\lambda dx} = \frac{2\pi\alpha}{\lambda^2} \left(1 - \frac{1}{\beta^2 n^2}\right) \tag{1.3}
\]

Where \(\alpha\) is the fine-structure constant. The amount of Cherenkov light is proportional to the distance traveled by the particle while its energy is above \(E_{th}\). It is then possible to relate the Cherenkov light emitted by the shower to the energy of the primary particle: for a gamma initiated shower approximately \(10^6\) Cherenkov photons between 300 nm and 600 nm are produced per TeV of primary energy. A significant fraction of emitted photons is lost due to scattering and absorption in air, see Figure 1.4. The main processes involved are Rayleigh and Mie scattering, the first dominating in good weather condition, the latter caused by water droplets and dust particles. Ozone absorption plays a very important role at wavelength below 340 nm [9] while \(\text{H}_2\text{O}\) and \(\text{CO}_2\) absorption dominates for \(\lambda \gtrsim 800\) nm [12]. The majority of the Cherenkov light reaching the ground is contained inside the wavelength interval \([300, 600]\) nm.

Cherenkov light emitted by a single vertical particle at a certain altitude arrives on the ground in a ring. The emission from all particle in a vertical \(\gamma\)-ray-induced EAS will appear on the ground in a circular, almost homogeneous distribution of photons referred to as the Cherenkov light pool, see Figure 1.3b. For a vertical shower from a 1 TeV \(\gamma\)-ray the light pool will be roughly 250 m in diameter with a density of \(\sim 100\) photons m\(^{-2}\) in the wavelength interval \([300, 600]\) nm [11].

Because the refractive index of air is close to unity, the difference in the speed of shower particle and Cherenkov photons is small. As a consequence, Cherenkov photons will almost follow the shower particles. At ground level shower particles are distributed in a curved front (shower front) of roughly one meter thickness. Most of the Cherenkov light arrives on the ground within a few nanoseconds.

### 1.2 Cherenkov telescopes design

As a result of an atmospheric shower a region on the ground is uniformly illuminated by a brief flash of Cherenkov light. A telescope placed anywhere inside this region can then image the shower. The collection area of the detector is therefore given by the surface area of the light pool, \(\sim 50 \cdot 10^4\) m\(^2\). Cherenkov telescopes uses mirrors to focus Cherenkov photons onto an array
of photon counting devices placed in the focal plane of the telescope, see Figure 1.5. IACT analysis allows to reconstruct all the relevant information about the event from properties of the recorded image of the Cherenkov light emission of the shower.

Detection of the Cherenkov emission of an EAS over fluctuations in the much brighter night sky background is possible thanks to its relative intensity in a short timescale and its small angular size. IACTs need large mirrors to collect enough Cherenkov photons and fine pixeled cameras with nanosecond electronics to reduce the integration of the night sky noise to the minimum. The mirrors of IACTs are usually made of small facets of $\sim 1$ m diameter arranged to form a unique reflective surface of the order of $\sim 100$ m$^2$. The camera is hold on the focal plane of the optical system by a stiff frame. The field of view (FoV) of such instruments must be large enough to exploit as much of the light pool area as possible, i.e a vertical shower 100 m away from the telescope must still be visible. Typical IACTs have a field of view of the order of $3.5^\circ$ [3]. To resolve shower details, pixel sizes of the order of $\sim 0.1^\circ$ are needed, resulting in cameras that are composed roughly one thousand photosensors. In IACTs low noise, high gain, linear photon counting devices with a fast ($\sim 2$ ns) rise-time are required. Despite progress in solid state photon detectors, currently only PMTs meet these requirements and are therefore employed in every IACT experiment.
Figure 1.5: The Whipple 10 m γ-ray Cherenkov telescope [13]. This telescope, operating from 1968 till 2012, pioneered the imaging atmospheric Cherenkov technique. The main features of IACTs are clearly visible. The ∼10 m mirror is composed of small facets. A four armed structure holds the camera in the focal point of the mirror. The camera is composed of 379 PMTs and have a field of view of ∼2.6°. Counterweights are placed on the back of the mirror to reduce the torques on the elevation mechanism.

All current IACT experiments use multiple telescopes to image the same shower from different positions. The spacing between the telescopes must be large enough to provide sufficient baseline for stereoscopic measurement, but small enough to fit multiple telescopes within the light pool of the same shower. Typical spacings are of the order of ∼100 m. The main advantages of the stereoscopic approach over single telescope observation are [2]: increased effective area, enhanced sensitivity, improved reconstruction of shower geometry and the possibility of applying coincidence requirements between telescopes. These coincidence requirements allows to reject at hardware level single telescope triggers due to local CR muons or night sky background. Each telescope can then operate with a lower camera trigger threshold thus lowering the energy threshold of the array of telescopes.

1.3 Data analysis

Analysis of IACT data can be used to recover all the relevant informations about the event, its arrival direction and energy, from the properties of the
shower images. Using the image’s shapes a large fraction of the hadronic CR induced background can also be rejected.

**Shower imaging**

The basic geometry of shower imaging is depicted in Figure 1.6. If the axis of the shower coincides with the optic axis of the telescope a circular image in the center of the camera will be produced. Showers whose axis is parallel to the optic axis will result in roughly elliptical images whose width and length corresponds respectively to the lateral and longitudinal development of the shower. The image axis will point to the region in the camera field of view where the shower originated.

![Figure 1.6: The Cherenkov emission along the shower axis is mapped in the focal plane of a Cherenkov telescope. Light produced higher in the atmosphere will be mapped closer to the camera center than light emitted at the end of the shower. [9]](image)

As a first step of the analysis an image cleaning algorithm is applied with the aim of retaining only the pixels that contain actual Cherenkov light from the shower, discarding the ones that are triggered by night sky background fluctuations. Image parametrization is then performed. The standard tech-
nique for image parametrization was introduced by Hillas [14]. Hillas parameters describe the position, shape and orientation of the images, assumed to be elliptical, in the camera.

Stereoscopic IACT systems allows for a three-dimensional reconstruction of the shower geometry, see Figure 1.7. The direction of the incoming $\gamma$-ray is reconstructed projecting all the images recorded for the same shower in a common angular coordinate system. The arrival direction of the shower in this reference frame is located at the intersection of the major axis of the images (Hillas axes). The point in the sky where the shower originated is then obtained by including information on telescopes’ pointings, geographic location and event time. To compute the impact point of the shower on the observation level (shower core) the images are projected in a planar coordinate system and the intersection of the major axes of the images is calculated. The precision of the shower geometry reconstruction increases with the number of images of the same shower that are available. With current instruments, a typical angular resolution is $0.1^\circ$ or slightly better for single $\gamma$-ray, but sufficiently intense point sources can be located with a precision of 10–20 arc seconds [15].

![Figure 1.7: Diagram illustrating the principles of shower geometry reconstruction. A) reconstruction of the shower core position. The two images are projected on a common plane at the observation level and the intersection of the major axes of the images is computed. B) The shower direction is obtained in a similar way; this time the images are projected in a common angular coordinate system. [16]](image)

The total amount of charge in the image (image size) is determined by the
amount of Cherenkov light that has been collected. This quantity depends on shower energy and on the distance of the shower maximum to the telescope. Once the geometry of the shower is reconstructed the energy of the primary can be obtained confronting the size of the image, or the average size if two or more telescopes have triggered, with lookup tables generated with MC simulations for different shower geometries. IACTs energy resolution is typically of the order of 20%.

**CR background rejection**

CR flux is $\sim 1000$ times higher than the strongest gamma ray flux [3]. Achieving effective CR rejection is therefore one of the greatest challenge for IACT experiments. Gamma-hadron discrimination is done by exploiting the main differences between the two kind of events: image shape and arrival direction.

While the flux of charged CR is isotropic, $\gamma$-rays are not deflected by cosmic magnetic fields and point to the source. Selecting events which came from the know position of the $\gamma$-ray source will minimize the acceptance of the telescopes for isotropic CR. A good angular resolution is therefore important to provide better background rejection capabilities.

As seen in Section 1.1, hadronic showers are more irregular in shape then electromagnetic ones. As a consequence, images from hadronic showers are on average longer and wider compared to those from $\gamma$-rays of equivalent energy, see for example Figure 1.8.

![Figure 1.8: Differences between images of a simulated 1 TeV $\gamma$-ray and a 2.6 TeV proton initiated showers. [17.]](image)
Longitudinal and transverse dimensions of the images will differ in the two cases. An effective way to quantify these differences is to compare each image with the expected image for a $\gamma$-ray event of equivalent energy and geometry, as obtained from MC simulations. Differences between recorded and $\gamma$-ray MC image width and length are expressed through the use of two parameters, the Mean Reduced Scaled Width (MRSW) and the Mean Reduced Scaled Length (MRSL) [9]. Each one of these is a measure of how much the image deviates from a “typical” $\gamma$-ray shower image.

Stereoscopic systems, with their improved angular resolution and the possibility of imaging the shower from different perspectives, provide better efficiency in gamma-hadron discrimination. Applying cuts on MRSW and MRSL and shower arrival direction background rejection factors of $\sim 100$ are achieved.

Residual background estimation

Complete background suppression is not possible. The number of non-$\gamma$-ray events miscounted as $\gamma$-ray events (residual background) must be then estimated. The simplest way to accomplish this task is to support observations of the source (ON runs) with observations of sky regions where no known gamma-ray source are present (OFF runs). These observations can be carried out while the telescope is pointing at a source selecting an empty region of the field of view (OFF region), or there could be dedicated observation during which the telescope is pointed to an empty sky field. A detailed description of the most common method used for background measurement can be found in [18].

1.4 Current Status of IACT

Observations with IACTs effectively began in 1989, with the first detection of a the Crab Nebula at TeV energies performed by with the 10 m Cherenkov telescope of the Whipple Observatory [13]. Since then a number of other experiments have followed and each new generation has increased the number of known VHE $\gamma$-ray sources by roughly an order of magnitude. An historical review of IACTs can be found in [19]. With the current third generation of IACTs the number of VHE $\gamma$-ray sources has risen to $\gtrsim 150$; of these around 100 are galactic sources while $\sim 50$ are extragalactic sources, mostly belonging to the Active Galactic Nuclei (AGN) class. A review of the status of VHE gamma astronomy can be found in [2, 3, 20].

As of today, the three most sensitive IACTs are H.E.S.S. [21], MAGIC [22]
Chapter 1.

and VERITAS [23]. Sensitivity is one of the most important parameter used to quantify the performance of a $\gamma$-ray observatory. It quantifies the ability of detecting a weak gamma ray source over the large CR background. Traditionally the integral sensitivity is quoted: the integral sensitivity at a given energy is defined as the minimum flux above that energy that can be detected in a given amount of time. Sensitivity of IACTs are generally referred to 50h of observing time. In Figure 1.9 the integral sensitivity of these instruments is presented. For comparison, the integral sensitivity of other non IACTs $\gamma$-ray observatories, Fermi LAT and HAWC, are presented in the same plot along with the expected sensitivity for CTA, the proposed next generation IACT.

![Figure 1.9: Comparison of the integral sensitivity for CTA (array layout E) and other VHE gamma ray experiments. VERITAS integral sensitivity is very similar to the HESS one and is not reported here. For CTA, MAGIC II and HESS the curve is referred to 50h of observation. One year is assumed for Fermi and HAWC. [15]](image)

Principal characteristics of three currently operating IACT instruments are presented in Table 1.1. In this table the sensitivity is expressed in Crab unit. The Crab nebula is in fact one of the brightest steady sources in the VHE $\gamma$-ray sky and often taken as a standard candle in TeV astronomy. Crab flux above 100 GeV is of the order of $9 \cdot 10^{-10}$ cm$^{-2}$s$^{-1}$.

**HESS** is an array of five telescopes located in Khomas highlands in Namibia. The array is composed of four 13m telescopes arranged in a square with 120m sides. Each of these telescopes is equipped with a $5^\circ$ FoV camera consisting of 960 pixels of 0.16°. A fifth large telescope is placed at the center of the square. This large telescope feature a parabolic mirror of 614 m$^2$ and a $3.5^\circ$ FoV camera consisting of 2048 PMTs of 0.07°. HESS operations started in

22
Figure 1.10: Three more sensitive IACT facilities currently operating December 2003 with the four 13m telescopes. The fifth telescope was added in July 2012, lowering the energy threshold of the system and improving its sensitivity. Thanks to the large FoV and being located in the southern hemisphere it has allowed for the first VHE survey of the Galactic plane, resulting in the discovery of $\approx 60$ new VHE Galactic sources [24].

**MAGIC** is a system of two large $236 \, \text{m}^2$ Cherenkov telescopes located in Roque de los Muchachos in the Canary Island of La Palma, Spain. MAGIC started operating as a single telescope system in 2003. In 2009, a second telescope of the same characteristics was added 85m away from the first. MAGIC telescopes were designed to achieve a low energy threshold. With an energy threshold of 25 GeV and being located in the northern hemisphere, this system is well suited for extra-galactic observations.

**VERITAS** is an array of four 12 m telescopes located at Fred Lawrence Whipple Observatory in Arizona, USA. The full array has started observations in 2007 while in 2009 the array layout was changed to increase the
Table 1.1: Principal characteristic of the three most sensitive IACT experiment currently operating. The sensitivity is expressed as the minimum flux (as a percentage of that of the Crab Nebula) above 1 TeV detectable at the 5\(\sigma\) significance level in a 50 h observation. [11].

Future development, The Cherenkov Telescope Array

Thanks to the success of the current generation of Cherenkov telescopes VHE \(\gamma\)-ray astronomy has become a well established astronomical domain. The number of known sources is growing rapidly but in many cases observations are not accurate enough to discriminate between different models. To acquire a deeper theoretical understanding of the processes governing the emission of VHE \(\gamma\)-rays a new instrument is required, more powerful than any conceivable upgrade of currents facilities.

The CTA (Cherenkov Telescope Array) Consortium is intended to develop such an instrument. CTA is a new generation IACT system and will provide significantly deeper insight into phenomena governing the VHE non-thermal universe. The scientific areas in which CTA is expected to contribute significantly are:

- **Cosmic Rays:** High energy photons can be produced in the interaction of energetic charged particles with the surrounding environment, matter or radiation fields. Observing galactic CR emitters in VHE gamma will provide deeper understanding of the physics mechanism governing particle acceleration near these objects [25].

- **Active Galactic Nuclei:** Supermassive black holes at the centers of active galaxies produce powerful outflows that offer excellent conditions for particle acceleration. CTA aims to detect a large number of such sources, allowing for the first time to conduct population studies of these objects. Measuring the spectra of a large sample of Active Galactic Nuclei (AGN) will also provide useful information to constrain the
theories of the extragalactic infrared and optical backgrounds as well as the intergalactic magnetic field [26].

- **Dark Matter and Lorentz Invariance Violation:** With its high sensitivity CTA could be able to detect features in diffuse photon spectra indirectly produced by dark matter annihilation. Its extended energy range will make it sensitive to a large range of dark matter particle masses. CTA will also provide new insight into other fundamental physics questions, such as searches for axion-like particles, effects of quantum gravity and violations of Lorentz invariance [27].

The CTA observatory will be composed of two sites, one for each of the northern and southern hemispheres, thus providing a full sky coverage. Each site will be composed of tens of Cherenkov telescopes of three different sizes. The main site will be in the southern hemisphere where the source rich galactic center region will be visible. Since emission up to PeV energies is expected from galactic sources, the southern observatory will be designed to have good sensitivity over four energy decades, from a few tens of GeV to above 100 TeV. The northern observatory will be dedicated to the observation of extragalactic sources. For these sources no emission at energies higher than \(\sim 10\) TeV is expected as the mean free path of VHE photons in the extragalactic medium decrease with energy [2]. CTA north will then be smaller than the southern site as coverage of the highest energies will not be mandatory.

CTA is designed to be 10 times more sensitive than any other IACT facility. In its core energy range, from about 100 GeV to several TeV it will reach milli-Crab (mCrab) sensitivity, a factor of \(10^4\) below the highest fluxes measured so far in bursts from transient sources. Extrapolating from the luminosity distribution of known sources, an instrument this sensitive is expected to enable the detection of roughly a thousand new sources [15].

Being composed of tens of telescopes spread over an area much larger than the Cherenkov light pool, most of the showers will be imaged by many telescopes. Angular resolution in the arc minutes range are foreseen these kind of events, achieving a factor of 5 improvement compared to today’s Cherenkov telescopes.

For the first time in this field CTA will be operated as an open observatory. Beyond a base program, which will include for example a survey of the Galaxy and deep observations of previously discovered sources, observations will be conducted according to peer-reviewed proposal from the scientific community.

Proposed for in 2006, CTA started as a European initiative. Since then many other countries and institutions had join the project. CTA Consortium
consists now of over 1000 scientists and engineers in over 160 institutions in 27 countries from Europe, the Americas, Asia and Africa, uniting the main research groups in this field. At the moment of writing the project is in the middle of a three-year Preparatory Phase at the end of which (late 2014) a Technical Design Report will be delivered. The selection of the locations for both north and south observatories is expected before the end of this year (2013). A five years construction phase is planned to start in early 2015. First scientific results from a partially deployed array are expected in 2016.

The final design for CTA will be defined only at the end of the Preparatory Phase. Many design options are being investigated at the moment and compared thought the use of extensive MC simulations. Two MC productions have been completed as of writing the thesis; for this work we will refer to results and specifications used in the first of such productions (CTA MC Prod1).

1.5.1 CTA design concept

The sensitivity of a Cherenkov telescopes is dominated by different phenomena at different energies, see Figure 1.11. Uncertainties in background estimation and subtraction dominate the low end of the energy range, while in the high energy range statistical errors limit the sensitivity. In the intermediate energy range the main limitation comes from gamma-hadron discrimination. To meet the sensitivity requirements over the entire energy range different technical solutions must be implemented at different energies:

Low energy range ($\leq 100$ GeV): The Cherenkov light yield of EAS is proportional to the energy of the primary $\gamma$-ray. Very large light collecting surfaces are therefore needed to detect showers down to a few tens of GeV, where Cherenkov light pool densities are of the order of a few photons per square meter. Since event rates at these energies are relatively high, there is no need for a large effective area and the surface covered by this part of the array can be relatively small, of the order of $10^4$ m$^2$. CTA opted to use a small number ($\leq 4$) of closely placed large size telescopes (LSTs).

The current baseline design for these telescopes is somehow similar to the MAGIC design and will be optimized to achieve the lowest possible energy threshold. It will use a parabolic mirror of 23 m diameter and a $4.5^\circ$ FoV with $0.1^\circ$ pixels. The mirror will be mounted on a alt-azimuth mount moving on rails for azimuth movement. The desire to rapidly repoint the telescopes following Gamma-Ray Burst (GRB) alerts has led to the choice of a light carbon fiber structure. Slewing speed of $180^\circ$ in 20 s are planned for the LSTs.
Figure 1.11: Toy model of a Cherenkov telescope illustrating the main limitation at the sensitivity of the instrument: (purple) systematic uncertainties in background estimation, (red) electrons, (green) hadronic CR, (black line) statistical uncertainties. The minimum detectable flux is given as a fraction of the Crab Nebula flux per energy bin. [15].

**Core energy ([0.1, 10] TeV):** Improved performances in this energy range can be achieved with an array of ~30 medium size telescopes (MSTs) with diameter of \( \simeq 12 \) m and spacing of about 100 m. Improved sensitivity over existing IACTs will be achieved both by the increased effective area of this part of the array and by the higher quality of shower images obtained since each shower will be imaged by many telescopes, allowing for better background rejection.

MSTs will cover the core of CTA energy range. Since around 30 telescopes of this type will be deployed for each site simplicity, reliability and ease of maintenance are particularly important in this case. The most probable design features a 12 m spherical Davies-Cotton [28] mirror with a focal length of 16 m. The camera will be composed of 1500 pixels of 0.18\(^\circ\) for a field of view of \( \simeq 8^\circ\).

**High energy range (> 10 TeV):** For energies above 10 TeV the main limitation is the number of detectable \(\gamma\)-ray showers. Effective areas of the order of several square kilometers are required to extend the performance of CTA in this energy range. Shower with these energies have large light yield and can be detected from distances well beyond the 150 m radius of
the Cherenkov light pool. The telescopes that will be used to extend CTA sensitivity to the highest energies must therefore have a large field of view to avoid truncation in the images from distant showers. At the moment two possibilities are being considered: either a large number of small telescope with mirror area of a few square meters and spacing of the order of light pool radius, or a smaller set of larger 10 m$^2$ telescopes which can see shower from distances of the order of 500 m. Both these solutions are called small size telescopes (SSTs).

Cost per telescope is one of the key parameters governing the choice of SSTs design. In principle a scaled down and simplified version of the MST could be used. The large FoV requirements will however led to an increased cost for the camera. For this reason other options exploiting a dual mirror design allowing for a more compact and cheaper camera with smaller pixel, allowing for solid state photosensors to be used, are being investigated and prototyped at the moment, see Figure 1.14.

1.5.2 Array layouts and performances

Performance of the whole instrument not only depend on the technical implementation of each type of telescope, trigger logics and analysis techniques also plays an important role. Another key aspect to be considered is the composition and layout of the full array, i.e the number and type of telescopes and their position on the ground.

A promising configuration is represented by the so called Array E, com-
posed of 4 LSTs, 23 MSTs and 32 SSTs spread over an area of \( \approx 3 \text{km}^2 \). Even if not the definitive choice, it can be thought of as a good representative of the layout of CTA south. Exploiting three different kinds of telescopes Array E offers good balance between high and low energy performances and meets the performance requirements over the whole energy range while fitting into the 80 M€ construction cost. The position on the ground of the telescopes composing Array E is shown in in Figure 1.15.

Differential sensitivity for array E is plotted in Figure 1.16. It is computed from MC simulations as the minimum flux that ensure a significant detection (above 5% the background level, with 5\( \sigma \) statistical significance and at least ten events) in each energy bin. Five logarithmic bins per energy decade are
Figure 1.15: Telescopes layout for the Array E in a $3 \times 3$ Km$^2$ grid. Large circles identify the LSTs, mid-size circles the MSTs and small circles represents the SSTs. [15]

used. Contribution of the three different types of telescopes are also shown. For CTA north, the sensitivity at the highest energy will be reduced, as no SSTs are planned to be deployed in this case.

Figure 1.16: Differential sensitivity in units of Crab flux of CTA array E as computed from MC simulation assuming 50h of observing time. Contributions from different telescopes are shown. [29]

Being composed of a large number of telescopes, CTA can be operated in a wide range of configurations, see Figure 1.17: it can be used as a single instrument for in depth study of a single object, or it can be split into smaller subsets (sub-arrays), each one with performance comparable or better than any current IACT system, operating independently from one another, allowing for example the simultaneous monitoring of several potentially flaring
sources. Survey capability of CTA will also benefit from the large number of telescopes. Groups of telescopes can be operated pointing to adjacent fields in the sky, with slightly overlapping field of view, in order to increase the area surveyed per unit time by roughly an order of magnitude at the cost of a reduced sensitivity.

Figure 1.17: Possible operating modes of CTA: a) in depth observation of a small region in the sky with maximum sensitivity, b) combination of deep observations (tens of telescopes) and monitoring (few telescopes) of flaring sources, c) divergent pointing survey mode. [15]
Chapter 2

Simulation of VHE light curves from Active Galactic Nuclei

Active Galactic Nuclei (AGN) are compact regions at the center of some galaxies (active galaxies) whose luminosity greatly outshine the stellar emission of the rest of the host galaxy. AGN are divided in two main families according to their radio emission, radio-loud AGN and radio-quiet AGN, the latter being the most numerous group with about 90% of the known sources belonging to this class. Radio-loud AGN are a well established class of VHE \( \gamma \)-ray emitters accounting for about one third of all the cosmic sources detected in the VHE regime.

Strong variability of the emission is one of the key features of these kind of sources. Unpredictable flux variations makes them difficult targets for IACTs due the limited field of view and duty cycle of these instruments. In this work, a MC simulation has been set up to identify the most effective way to observe AGN with CTA. This has required the simulation of AG-like light curves, i.e time series that represent the temporal evolution of the flux of the source in a given energy band.

In Section 2.1 AGN will be described, presenting the main features of these objects. Section 2.2 will focus on the blazar class of AGN, representing the great majority of AGN which are detected at VHE. Section 2.3 is devoted to a presentation of the simulation of AGN light curves: first the observed properties of high energy (HE, \( \sim [0.1, 100] \) GeV) and VHE AGN light curves will be presented, then the details of the algorithm used to simulate AGN-like light curves will be described.
Chapter 2.

2.1 Active Galactic Nuclei

It is currently accepted that a super massive black hole of the order of \(10^6 - 10^9\) solar masses is present at the center of active galaxies. The enormous energy outflow of AGN is believed to be ultimately powered by the gravitational energy lost by matter accreting onto the black hole. This spiraling gas is arranged in an accretion disk where viscous phenomena are responsible for bright UV thermal emission. Strong optical and UV emission is also observed from gas clouds moving in the gravitational potential of the black hole, the so called broad line clouds (see Figure 2.1). Due to the high velocity of matter in the vicinity of the black hole, emission lines from this region are strongly Doppler shifted; shifted emission from gases moving towards and away from the observer then results in a broadening of the lines, hence the name. Slower moving gas clouds further away from the black hole produce emission with narrower lines (narrow line clouds). An opaque, dusty region, assumed toroidal, is present well outside the accretion disk and obscures emission from the broad line clouds and the disk itself along some lines of sight. Radio-loud AGN are characterized by the presence of two powerful collimated outflows of energetic particles (jets) streaming away from the black hole parallel to the axis of rotation of the system. Jet morphology has been resolved with high precision in the radio band using Very-Long-Baseline Interferometry (VLBI) techniques. These studies have revealed that AGN jets are often inhomogeneous with higher density regions traveling through the jet. These denser plasma regions moves at relativistic speeds providing the perfect environment to accelerate charged particles that can be responsible for the strong non-thermal emission observed in radio-loud AGN. If the population of emitting particles are relativistic then beaming [30] of the emitted radiation can occur, accounting for the high VHE luminosity observed in these objects.

AGN were originally separated into many different subclasses according to differences in their observed spectra. Those differences are now commonly believed to be caused by the strong anisotropy of AGN emission [31]. If the AGN axis form a small angle with the line of sight the emission will be dominated by the non thermal emission of the jets. AGN of this type are known as blazars. If the line of sight is roughly perpendicular to the disk axis, no emission from the disk or the inner broad line region will be visible, due to absorption from the dusty torus and only features from the narrow line region will appear in the spectra of these sources. In this category are included Seyfert type II galaxies. For intermediate angles the disk and the broad line region will also be visible (Seyfert I).
2.2 Blazars

Blazars are radio-loud AGN whose jets are closely aligned with the line of sight. Blazar spectra are therefore dominated by the Doppler boosted non-thermal emission from the jets. Blazar spectra span over almost 20 orders of magnitude, from radio to VHE. Thanks to their high luminosity in the VHE gamma waveband blazars constitute the great majority of extragalactic VHE $\gamma$-ray sources detected by IACT experiment despite representing the less populous of all AGN classes. At the moment 41 of the 45 extragalactic sources detected at VHE with IACT experiments belong to this class [26]. Blazars are divided into two sub-categories: BL Lacertae objects (BL Lacs) and flat spectrum radio quasars (FSRQs). FSRQs are observationally characterized by the presence of strong and broad emission lines in the optical band, which are weak or not present in the spectra of BL Lacs. Of the 41 blazars detected by IACTs, only three FSRQs have been detected so far at VHE [32].

The spectral energy distribution (SED) for these sources shows a characteristic double bump structure, with a low energy component peaking from radio to X-ray domain and a high energy component from X-ray to $\gamma$-ray, [33], see Figure 2.2. In the VHE band AGN spectra are generally well reproduced by a power law with index (photon index) ranging from 2 to 3. Physics at the

---

**Figure 2.1:** Diagram illustrating the main features of a radio-loud AGN. Surrounding the central black hole there is the luminous accretion disk. Broad emission lines are produced in clouds orbiting above the disk (black dots). The dusty torus obscures the broad line region and the disk from transverse lines of sight. Narrow emission lines are produced in clouds much further from the black holes (gray dots). Collimated jets of relativistic plasma emanate from the center. [31]
basis of AGN emission is still poorly understood. While the low-frequency emission almost certainly originates from synchrotron emission of the relativistic jet electrons in the ambient magnetic field, it is currently not clear whether the high-frequency part of blazars SEDs has leptonic or hadronic origin [35]. In a pure leptonic scenario the high energy peak is interpreted as relativistic electrons inverse Compton scattering on low energy ambient photons, either the same produced by synchrotron emission (Synchrotron-self-Compton models) or coming from other part of the AGN (External Inverse Compton models). If, however, a population of energetic protons is assumed, high energy photons can be produced in a variety of ways: electromagnetic decay of neutral pions produced in the interaction of energetic protons on ambient photon fields or synchrotron emission from protons, $\pi^\pm$, and $\mu^\pm$. In this case the second peak of the SED will have an hadronic origin, while leptons will still be responsible for the low-frequency bump; these kinds of models are called lepto-hadronic models.

**Blazar VHE variability**

Limited field of view and duty cycle of IACTs, as well as source visibility and weather constraints often result in a sparse time coverage of the source. Nonetheless, VHE variability has been observed at all time scales in 20 of the 45 known sources [26], see for example Figure 2.3. The temporal evolution
of a source flux can be depicted through the use of its light curve, representing the flux in a given energy band as a function of time. Many VHE AGN light curves appear aperiodic and flux variations down to the minute timescale have been observed for the BL Lacs Mkn 421, PKS 2155-304 and Mkn 501 [36], [37], [38]. The most extreme manifestation of AGN variability are flares. During a flare, a source can increase its flux by several orders of magnitude. One of the most famous and studied flare is arguably the PSK 2155-304 2006 flare shown at the bottom in Figure 2.4. On that occasion, the flux increased from the average value of about 15% of the Crab Nebula to approximately 7 times the Crab flux with peaks of almost 15 times the Crab, an increase of a factor 100 compared to the average. Thanks to the exceptional source’s brightness, data acquired during a flare are almost noise free.

Figure 2.3: PKS 2155-304 light curves as obtained by HESS. The source appears variable at all time scales. Top: 3-year monthly light curve. The flare occurred in July 2006 is clearly visible. Bottom: Nightly light curve recorded in August 2003. The dashed lines represent the best $\chi^2$ fit of the data to a constant, from [39].
Variability studies are essential in determining the physics of the central region of AGN. Determining characteristic time scales, spectral changes, correlations, and delays among variability at different energies provide crucial information on the nature and location of the emitting region. For example, a measure of the minimum variability time scales can be used to constrain, by means of a causality argument, the size (and location) of the VHE emitting region. As of today, variability at the shortest time scales has been measured only during states of high activity of the sources. With its large effective area CTA will have access time scales well below the possibilities of current instrument for an increased number of sources. Moreover, CTA’s large energy range will provide an improved tool to study possible correlations of the emissions at different energies, from which considerable insight on AGN $\gamma$-ray emission mechanism can be gained.

### 2.3 Simulation of AGN light curves

Currently the mechanism responsible for VHE AGN variability is not known. For this reason a phenomenological approach has been adopted, producing light curves that exhibit the same features as observed AGN VHE light curves. Like any time series, AGN light curves can be characterized by their
statistical properties such as mean and variance, as well as by their properties in the frequency domain, notably their power spectral densities (PSD). These tools have been used to match the features of the simulated light curves with real data.

Unfortunately very little information is available on temporal properties of AGN VHE light curves. Many of the detected blazars are sparsely covered by observations. Moreover, effective areas of current instruments do not allow for a sufficiently fine time binning of the sources flux. As a consequence, VHE blazar light curves usually contains few points and large gaps. Satisfactory statistical and Fourier analysis has only been possible for the few, bright, well studied sources as PKS 2155-304, which cannot be taken as representative of the whole class of objects. For this reason we have referred to data taken from other experiments in other energy regimes to complement the scarce information available in the VHE band. In particular, we will refer to results from the Fermi Gamma-ray Space Telescope [1]. Its main instruments, the Large Area Telescope (LAT), is an imaging high-energy $\gamma$-ray pair conversion telescope with an energy range from $\sim$20 MeV to $\sim$300 GeV. Thanks to its large FoV and higher fluxes in this energy regime, it had enabled the detection of about 700 AGNs. Referring to data taken in other energy domains we make the assumption that AGN light curves’ properties do not change significantly from the VHE to the HE band. This seems to be a reasonable assumption since the same part of blazar spectra, the high energy bump, is observed in both the high and very high energy band. Emission in these two adjacent energy bands therefore originated from the same physical phenomena, and it is then reasonable to assume that variability properties in these two energy bands will not differ significantly. In any case it is worth stressing that the simulated light curves are meant to only reasonably reproduce the observed properties of real sources; a detailed model of AGN VHE variability is beyond the scope of this thesis.

2.3.1 Characterizing VHE AGN variability

Many different models can be used to simulate AGN variability. We have chosen to use a method capable of producing light curves which feature two characteristics observed in AGN light curves: a power-law PSD and a linear relation between the root mean square (RMS) amplitude of fluctuations and the mean flux of light curves’ segments.

The PSD is one of the most common tool used to characterize the structure of a time series. PSDs of non periodic time series are continuous functions of the temporal frequency that represent the amount of variability power in the signal for each frequency. PSDs of AGN light curves are often described
Chapter 2.

Figure 2.5: Average PSD for the 156 brightest ($\phi(E > 100\text{MeV}) > 3 \times 10^{-8}\text{cm}^{-2}\text{s}^{-1}$) BL Lacs (blue) and 56 FSRQs (red) present in Fermi second AGN catalog. The PSD is normalized to fractional variance per frequency unit. [40]

as power laws:

$$P(f) \propto f^{-\beta}$$

(2.1)

where $P(f)$ is the power at frequency $f$, over a wide range of frequencies. The PSD of any real process must flatten out at the lowest frequencies in order to avoid a divergence in the power in the signal, i.e $\beta \leq 1$ at sufficiently low frequencies. This flattening has not yet been observed at high energies suggesting that the time scales at which this occurs are longer than the ones currently probed by observations.

Fourier spectral analysis has been performed by Fermi for a sample of 217 blazars (156 BL Lacs and 56 FSRQs) in the second LAT AGN catalog [40]. In this case monthly binned light curves of the integral flux above 100 MeV are used. The average PSD are shown in Figure 2.5. The power law indexes are about 1.15 for both sources families in the frequency range [0.033, 0.5] month$^{-1}$. Similar analysis performed on a smaller sample of bright sources (22 FSRQs and 6 BL Lacs) using three- and four-days binned light curves, lead to values of $\beta$ of 1.7 and 1.5 for BL Lacs and FSRQs respectively [41]. In the VHE energy domain the only case in which Fourier analysis of the light curve has been performed is the PKS 2155-304 2006 flare observed by HESS. The PSD for the 1 minute light curve shown in Figure 2.4 has been computed and is shown in Figure 2.6. The data are compatible at
90\% confidence level with a power law PSD with index $\beta = 2$ in the frequency range $\sim [10^{-4}, 10^{-3}]$ Hz [37].

![Power spectral density for the 1 minute PKS 2155-304 light curve obtained with HESS during the night of MJD53944. The gray area corresponds to the 90\% confidence interval obtained from simulated light curve with a PSD $\propto f^{-2}$. [37].](image)

Figure 2.6: Power spectral density for the 1 minute PKS 2155-304 light curve obtained with HESS during the night of MJD53944. The gray area corresponds to the 90\% confidence interval obtained from simulated light curve with a PSD $\propto f^{-2}$. [37].

The other feature observed in AGN light curves that needs to be reproduced in the simulated light curves is the linear relation between the RMS amplitude of flux variation and the mean flux computed along the light curve. This proportionality implies that fluctuations around the mean value, i.e. variability, are enhanced when the flux is higher. The RMS amplitude $\sigma_{RMS}$ of a set of $N$ discrete points $x_i$ is defined as the square-root of the variance $\sigma^2$ of the set:

$$\sigma_{RMS} = \sqrt{\sigma^2} \quad \sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^2 \quad (2.2)$$

where $\overline{x}$ indicates the arithmetic mean of the set. It has been noted that the $\sigma_{RMS}$ of segments of the observed light curves varies randomly around a mean value which scales linearly with the mean flux of the segment. The RMS amplitude averaged over a group of contiguous light curve segments is then proportional to the average mean flux of the segments. This fact has been observed in the X-ray band for many AGN and other accreting objects, like binary systems [42]. In the VHE domain such a behavior has been detected in the well studied PKS 2155-304, see Figure 2.7, [43].
Chapter 2.

Figure 2.7: Excess RMS vs \( \langle \phi \rangle \), mean integral flux above 200 GeV for observation of PKS 2155-304. The excess RMS is defined as the square root of the variance of the light curve subtracted the measurement error \( \sqrt{\sigma^2 - \sigma_{err}^2} \). Full circles represent observation during MJD 53944–53947. Open circles are additional point obtained extrapolating flux back to 200 GeV from measurements that had an higher energy threshold. The RMS and \( \phi \) are computed for segments of 80 minutes extracted from the four-minute binned light curve. The dotted line represent a linear fit to the points. [43].

The PSD shape and the linear RMS-flux relation are fundamental properties governing the time structure of the light curve. The mean and variance set the scales of the flux values and fluctuations. The variance of a light curve is more commonly expressed through the use of the fractional variance \( F_{\text{var}} \), defined as the ratio between RMS amplitude \( \sigma_{RMS} \) and the mean flux \( \phi \):

\[
F_{\text{var}} = \frac{\sigma_{RMS}}{\phi} = \frac{\sqrt{\sigma^2}}{\phi} \tag{2.3}
\]

Fractional variances ranging from 20% to 60% have been observed in the VHE light curve above 200 GeV of PKS 2155-304 [43]. Values of \( F_{\text{var}} \) for most of the sources in Fermi’s second catalog of Active Galactic Nuclei are found to vary among 20% and 80% [40]. Mean fluxes in the VHE regime are of the order of some percentage of the flux of the Crab nebula (about \( 9 \times 10^{-10} \text{cm}^{-2} \text{s}^{-1} \) above 100 GeV), reaching \( \sim 10\% \) of the Crab for the brightest sources such as Mkn 421, PKS 2155-304.
Chapter 2.

2.3.2 Light curve simulation algorithm

The algorithm used to generate light curves featuring a power law PSD and linear relation between the mean flux and the RMS amplitude is based on the method proposed in [44]. In this paper it has been observed that the linear RMS-flux relation observed in AGN light curves represent a strong constraint on models that must be used to reproduce AGN light curves. In particular, it is demonstrated that only non-linear models\(^1\) of AGN variability are able to produce that relation. According to the method proposed in that paper, a non-linear light curve, which will therefore feature the linear RMS-flux relation, can be obtained through an exponential transformation of a linear light curve. That is, each point of the light curve \(l(t_k)\) can be obtained taking the exponential of a linear, aperiodic light curve with zero mean, \(x(t_k)\):

\[
l(t_k) = \exp[x(t_k)]
\]

(2.4)

This exponential transformation accounts for the non-linearity of this model as it can be easily recognized by considering the series expansion of the exponential function.

To generate the linear light curves \(x(t_k)\), the algorithm proposed by Timmer & König [45] has been used. This algorithm produces aperiodic time series with a power law PSD \(P(\omega) \propto \omega^{-\beta}\), where \(\beta\) is arbitrary slope. The Timmer & König method is based on a result of the theory of spectral estimation that states that the Fourier amplitude \(F(\omega)\) corresponding to the angular frequency \(\omega\) for a process with an underlying PSD \(P(\omega)\) is a complex Gaussian random variable that can be written in the form:

\[
F(\omega) = \mathcal{N}(0, \frac{1}{2}P(\omega)) + i\mathcal{N}(0, \frac{1}{2}P(\omega))
\]

(2.5)

where \(\mathcal{N}(\mu, \sigma^2)\) represent a random number drawn from a Gaussian distribution of mean \(\mu\) and variance \(\sigma^2\). For a discrete time series of \(N\) points and length \(T\), the Fourier transform is also discrete and there are \(N\) Fourier amplitudes \(F(\omega_i)\) corresponding to angular frequencies \(\omega_i\) given by:

\[
\omega_i = \frac{2\pi}{T} i \quad i = -\frac{N}{2} + 1, ..., \frac{N}{2}
\]

(2.6)

For each of these frequencies the respective Fourier components are computed according to Equation 2.5. To obtain a real-valued time series must

---

\(^1\)A linear model is one in which the output (here the light curve’s points) is a linear combination of some input variables, such that multiplying the inputs by a constant multiplies the outputs by the same constant. A model that does not conform to this definition is said to be non-linear.
be $F(-\omega_i) = F^*(\omega_i)$: only the Fourier amplitude corresponding to positive frequencies need to be computed. Setting $F(0) = 0$, we obtain a light curve with zero mean. If $N$ is even, then due to symmetry $F(\omega_{N/2})$ is real and only one random number is needed. Once the discrete Fourier transform of the signal is constructed the time series $x(t_k)$ can be obtained by taking the inverse Fourier transform (Fast Fourier Transformations can be used):

$$x(t_k) = \sum_{i=-N/2+1}^{N/2} F(\omega_i)e^{-i\omega_i t_k} \quad k = 0, ..., N - 1 \quad (2.7)$$

From this relation the linearity of this model with respect to the inputs (here the $F(\omega_i)$) is evident. Figure 2.8 shows two example of time series obtained with this method using PSD slopes $\beta = 1$ and $\beta = 2$. We have checked that the light curves produced with this method show the right PSD shape. The PSD is an underlying property of the process and can be estimated for each realization, i.e each light curve, through the use of the periodogram, defined as $\text{Per}(\omega) = |F(\omega)|^2$. The periodogram for the two light curves of Figure 2.8 is also shown. Fits with power law functions show good agreement with the input PSD slopes.

Figure 2.8: Light curves generated with the Timmer & König algorithm and their periodograms. Left) Input PSD slope of 1 (flicker noise), periodogram fit result $1.01 \pm 0.02$. Right) Input PSD slope of 2 (random walk noise), periodogram fit result $1.98 \pm 0.02$
Chapter 2.

The Timmer & König algorithm is able to produce light curves with the desired power density spectra. Being a linear model, the exponential transformation is needed to reproduce the RMS-flux relation. It must be noted that this exponential transformation does alter the shape of the PSD of the Timmer & König light curve. However, for the broad continuous PSD observed in AGN, the distorting effect introduced by the exponential transformation is relatively small, as demonstrated in Appendix B of [44]. In Figure 2.9 we plot the distributions of PSD indexes estimated by fitting the periodogram of 1000 linear and non-linear light curves of \(10^5\) points each. The agreement between the two distribution is considered satisfactory. It is therefore reasonable to neglect the slight distortion of PSD slope introduced by the exponential transformation and use the observed PSD index as input of the linear light curve.

![Figure 2.9: Distribution of PSD indexes obtained fitting the periodogram for 1000 linear light curves (black) and the corresponding exponential light curves (red). The light curves have \(10^5\) points and the input PSD slope for the Timmer & König algorithm is 2. The mean of both distribution is 2 and the RMS is 0.006 and 0.008 for the linear and non-linear light curves respectively.](image)

An example of a non-linear light curve obtained with this method is shown in Figure 2.10, together with the Timmer & König light curve from which it was created. Compared with the original linear light curve, the new one exhibits much more pronounced fluctuations as points above the mean (zero) are enhanced while points below the mean are reduced. Consequentially when variations from the mean are larger, i.e when the variance of the linear light curve is increased, flares in the non-linear light curve are more strongly exaggerated compared to the dips. The effect of variation in the variance of the linear light curve is shown in Figure 2.11, where three exponential light
Chapter 2.

Figure 2.10: Linear light curve (top) and the light curve obtained through the exponential transformation of Equation 2.4. The linear light curve has been generated with a PSD index of 2. Before making the transformation the fractional variance of the linear light curve has been set to 80%.

curves are obtained from the same linear light curve. Even if the temporal structure is the same for all of them, the fractional variance of the linear light curve has been changed before taking the exponential transformation. We have to keep in mind that the appearance of our simulated light curve is strongly influenced by the fractional variance of the linear light curve. This parameter is inherent in our simulation algorithm but has no clear physical interpretation. We will call this parameter non-linearity index $I_{Nlin}$.

We have checked that our light curve features the correct linear relationship between the RMS amplitude and the mean flux. For this, a light curve of $10^5$ points has been generated and the RMS and mean flux are computed for light curve segments of 20 points each. The average RMS amplitudes, $\langle RMS \rangle$, and mean fluxes, $\langle \phi \rangle$, are computed for groups of 50 of these segments. The resulting plot of $\langle RMS \rangle$ vs. $\langle \phi \rangle$ is shown in Figure 2.12, where the expected proportionality between the two quantity is clearly visible.

As can be seen from Eq. 2.6 the length $T$ and frequency $\nu = N/T$ of the light curves that will be simulated define the range of angular frequencies that will contribute to the light curve’s variability:

$$\omega_{\text{min}} = \frac{2\pi}{T}, \quad \omega_{\text{max}} = \frac{\pi N}{T} = \pi \nu$$  \hspace{1cm} (2.8)
Figure 2.11: 3 non-linear light curve of $10^4$ point obtained from the same linear light curve but with different values of $I_{Nlin}$: $I_{Nlin} = 20\%$ (top), $I_{Nlin} = 50\%$ (middle), $I_{Nlin} = 80\%$ (bottom). Note the change on the y-axis scale. The PSD index for the original linear light curve is 2.
Figure 2.12: $\langle RMS \rangle$ vs. $\langle \phi \rangle$ computed for a non-linear light curve of $10^5$ point (see text for details). A fit with a straight line yields $\chi^2/\text{ndf} = 0.0001/98$.

Length and frequency of the light curves are chosen in order to include contributions from all the frequency components that are currently probed by observations in the HE and VHE domains. The lowest frequency we will consider is $0.033$ month$^{-1}$, probed by Fermi for sources in the second catalog of AGN detected by the LAT [40], while the highest frequency will be $1$ minute$^{-1}$, as in the case of PSK 2155-304 2006 flare light curve measured by HESS. We will then extend the PSDs measured by these two instruments to cover in this frequency range. The PSD measured by the Fermi telescope will be extrapolated towards the highest frequencies while the one measured by HESS will be extrapolated towards the lowest frequencies.

To cover this frequency range, light curves 30 months long with one point per minute were generated. Light curves that will be used in this work can be much shorter, reflecting the fact that IACTs have roughly only a thousand hours per year of available observing time. From the 30 months light curves segments of 500h-lengths will be extracted at random position. In this way the light curves will have a duration that is comparable with the observing time per year available to IACTs, while still including variability from all the frequency components that has been observed.

Once the correct temporal structure of the light curve is obtained the mean and the fractional variance can be easily tuned to match the properties of real AGN data. The desired fractional variance $F_{\text{var}}$ is set by applying the
following transformation to each point of the light curve $l(t_k)$:

$$\phi'_{LC}(t_k) = \frac{F_{\text{var}} \phi_{LC}}{\sqrt{\sigma_l^2}} l(t_k)$$

(2.9)

where $\phi_{LC}$ is the desired value for the mean flux of the simulated light curve and $\sigma_l^2$ is the variance of the unnormalized light curve $l(t)$. The mean can be set to $\phi_{LC}$ via:

$$\phi_{LC}(t_k) = \phi'_{LC}(t_k) - \phi_{LC} + \phi_{LC}$$

(2.10)

To conclude, in this section we summarize the steps of the algorithm used to generate light curves with similar characteristic as real AGN data. The input parameters needed are also listed in Table 2.1.

1. Generate a long linear light curve of 30 months with one point each minute with the Timmer & König algorithm. This light curve must have the same PSD slope of the desired light curve.

2. Extract segments of the desired length $T = 500h$ at random positions along the long 30 months light curve.

3. Set the fractional variance (Equation 2.9) of the 500h linear light curve segments to match $I_{\text{Nlin}}$. Add an arbitrary offset and normalize this light curve to its mean.

4. Generate non-linear light curves via the exponential transformation of Eq. 2.4.

5. Set the desired fractional variance and mean using Eq. 2.9 and 2.10.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>$T_{LC}$</td>
<td>s</td>
</tr>
<tr>
<td>Frequency</td>
<td>$\nu$</td>
<td>Hz</td>
</tr>
<tr>
<td>PSD index</td>
<td>$\beta$</td>
<td>adim.</td>
</tr>
<tr>
<td>Mean flux</td>
<td>$\phi_{LC}$</td>
<td>cm$^{-2}$s$^{-1}$</td>
</tr>
<tr>
<td>Fractional variance</td>
<td>$F_{\text{var}}$</td>
<td>adim.</td>
</tr>
<tr>
<td>Non linearity index</td>
<td>$I_{\text{Nlin}}$</td>
<td>adim.</td>
</tr>
</tbody>
</table>

Table 2.1: Input parameters needed by the light curve simulation algorithm.
2.3.3 Simulated light curves parameter space

With this algorithm light curves with different properties can be generated. We will consider variations of mean flux, fractional variance, PSD index and non-linearity index. The length and frequency of the light curves will be kept constant at to 500 h and $1/60$ Hz respectively. For the sake of simplicity, light curves’ parameters are varied one at the time around their “reference” values. Reference values and considered variations for these parameters are presented in Table 2.2.

The reference light curves will be characterized by a PSD index $\beta = 1.15$, the value measured by the Fermi telescope for the sources in the Second LAT AGN Catalog [40]. Other values of $\beta$ that will be considered are $1.7$ and $2$, as measured by Fermi for the brightest BL Lacs, and by HESS for PKS 2155-304 during its 2006 flare. Fractional variances for AGN light curves in the high and very high energy regimes spans from $\sim 20\%$ to $\sim 80\%$; the reference value, $F_{\text{var}} = 50\%$ is chosen roughly in the middle of this range and close to the values measured by HESS on PKS 2155-304; variations of $\pm 30\%$ around this value will be considered. The reference value for the non-linearity index is arbitrarily set to $50\%$. Considered variations of this parameter are $I_{\text{Nlin}} = 20\%$ and $I_{\text{Nlin}} = 80\%$. The reference value for the light curves’ mean flux is $1\%$ of the Crab nebula flux above $100$ GeV. Sets of light curves with mean fluxes of $0.5\%, 3\%, 10\%$ and $30\%$ of the Crab have also been produced. For mean fluxes of $1\%, 3\%$ and $10\%$ of the Crab seven sets of light curves have been produced in order to take into account the considered variations of PSD index, fractional variance and non-linearity index. Two other sets of light curves have been produced with mean fluxes of $30\%$ and $0.5\%$ of the Crab and the other parameters set to their reference value. Thus twenty-three different sets of light curves have been produced, each one containing 1000 light curves.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reference value</th>
<th>Variations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.15</td>
<td>1.7, 2</td>
</tr>
<tr>
<td>$\phi_{\text{LC}}$ (% Crab)</td>
<td>1</td>
<td>0.5, 3, 10, 30</td>
</tr>
<tr>
<td>$F_{\text{var}}$</td>
<td>0.5</td>
<td>0.2, 0.8</td>
</tr>
<tr>
<td>$I_{\text{Nlin}}$</td>
<td>0.5</td>
<td>0.2, 0.8</td>
</tr>
</tbody>
</table>

Table 2.2: Reference light curves’ parameters and their considered variations.
Chapter 3

Observing variable sources with CTA

Low duty cycle and limited field of view are the main disadvantages of IACTs. The ability of Cherenkov telescopes to successfully observe sources as variable as AGN is particularly limited by these factors. For weak sources flares are often the only occasion when their flux is above the sensitivity threshold of the experiment. These sources appear transient with current instruments as they are only visible during periods of high activity. Observing transient sources with Cherenkov telescopes is then a fortuitous event since it requires the source to flare while being in the small field of view of the instrument. To account for this fact IACTs usually rely on other instruments observing the sky at longer wavelengths to know if the source is flaring and could therefore be successfully observed. Correlations among AGN emissions at different wavelengths have been observed (see for example [46]). These correlations suggest the coincidence of flares at different energies. However VHE flares with no counterparts in other wavelengths, so-called orphan flares, have also been observed, as in the case of the AGN 1ES 1959+650 [47]. In that occasion the flux at TeV energies increased by a factor of 10 with no noticeable increase in the X-ray. In the case of strong AGN, which are always within the sensitivity reach of the instruments, monitoring campaigns are usually performed to maximize the probability of observing a flare. In these cases the source is observed often but for only short intervals of time, extending the observations only in case there is some hint that a flare is approaching. It is evident that both approaches are not able to guarantee the success of the observation.

The work presented here is intended to be a first quantitative approach to the optimization of CTA observing strategies for variable sources, namely AGN. The goal is to find, among all the possible ways in which CTA can be
Chapter 3.

operated, the most successful in accomplishing a given task. In particular, optimization of observing strategies will be performed with respect of two different scenarios: fastest way to detect a source and monitoring.

Observing strategies describe the way in which the instrument is used. They are defined by the sequence of time intervals in which the source is pointed at by the instrument (pointing scheme) and by the particular sub-array used. Many different CTA sub-arrays can be used to accomplish the same task in different amounts of time, i.e. a small sub-array with little effective area will require more time to achieve the same significance compared with a bigger one. The optimal observing strategy will then result from a balance between the number of telescopes participating in the observation and the time the observation requires.

Observing strategies employing different sub-arrays and pointing schemes have been modeled. A simulation has been set up to apply the observing strategies to the observation of the simulated AGN-like sources. The performances of each observing strategy will be evaluated in terms of the probability of detecting the source.

In Section 3.1 will be presented the model of the observing strategies that has been developed. Section 3.2 will be devoted to a description of the method used to evaluate the performance of the observing strategies.

3.1 Implementation of the observing strategies

A simple model of observing a source with CTA has been set up. The two elements defining an observing strategy for CTA, the sub-array and the pointing scheme, are included in this model. Pointing schemes represent the way in which the observing time is spread along the light curve. The sub-array used to carry out the observations will be characterized through the use of its response functions for effective area and background rate.

A pointing scheme specifies the series of time intervals (observing windows) in which the telescopes are pointing at the source. This sequence can be defined specifying the prescription (observing mode) used to place the observing windows along the light curve, by the total amount of time available for observation ($T_{\text{obs}}$), and by the duration of each windows. Only pointing schemes with a constant window length ($T_{\text{win}}$) will be considered: all the windows will have the same duration.
An observing strategy is then defined by five quantities:

\[
\begin{align*}
\text{Observing strategy} & \quad \{ \text{Sub-array} \} \quad \{ \text{Effective Area} \} \quad \{ \text{Background rate} \} \\
& \quad \{ \text{Pointing scheme} \} \quad \{ \text{Observing mode} \} \quad \{ \text{Total observing time } T_{\text{obs}} \} \\
& \quad \{ \text{Window length } T_{\text{win}} \}
\end{align*}
\]

### 3.1.1 Pointing schemes

We select segments of the simulated light curves representing time intervals in which the source is pointed at by the telescopes. In principle, a light curve extends through both days and nights. For a realistic simulation, observations should be taken only during night time, resulting in the waste of half of each simulated light curve’s points. The importance of using pointing schemes that take into account night-day alternation has been tested with a preliminary analysis. The performances of the observing strategies have been evaluated with and without considering the night-day sequence. No appreciable difference was found between the two cases. For this reason we choose to ignore the night/day alternation and use the entire length of the light curves to place the observing windows.

Two kinds of pointing schemes have been considered: periodic and random observations. In both cases the length \( T_{\text{win}} \) of the observing windows is fixed, the difference between the two observing modes lying in the prescription used to distribute the observing windows along the light curve. Since all windows have the same lengths, their sequence is defined by the series of their starting points \( t_{s}(i) \).

In the case of periodic observations the windows are placed 12 h apart, i.e. there is one window every night:

\[
t_{s}(i) = i \cdot 12 \text{ h} \quad t_{e}(i) = t_{s}(i) + T_{\text{win}}
\]

Starting from the beginning of the light curve at \( t_{s}(0) = 0 \text{ s} \), we continue to construct windows until the total duration of all windows is equal to the total observing time \( T_{\text{obs}} \). If necessary the last window is resized in order not to exceed \( T_{\text{obs}} \). It should be noted that in case of small \( T_{\text{win}} \) (or short light curves) it is possible that the light curve is not long enough to cover the available observing time.

In the random pointing scheme, the starting points of the windows are drawn from a uniform distribution extending from 0 to \( T_{LC} - T_{\text{win}} \):

\[
t_{s}(i) = U(0, T_{LC} - T_{\text{win}}) \quad t_{e}(i) = t_{s}(i) + T_{\text{win}}
\]
No overlap between the windows is permitted. As in the previous method, windows are constructed until the total observing time is covered and the last window is resized if needed.

### 3.1.2 Instrument response functions

Given the large number of telescopes comprising CTA, many different sub-arrays can be extracted. We restrict our analysis only to sub-arrays composed exclusively of MSTs. SSTs’ high energy threshold make them unsuitable for observations of extragalactic sources. Among LSTs and MSTs, the latter are more numerous and will play a major role in sub-arrays’ composition. As representative of the different types of sub-arrays of MSTs that can be extracted from the full CTA array (array layout E), we will then consider three cases: a small sub-array composed of 4 MSTs arranged in a square with sides of 120 m, a medium sized sub-array of 9 MSTs placed in a square of 340 m side with one telescope in the middle and a large sub-array composed of all MSTs present in array E. For this analysis the only relevant informations on the sub-arrays are the effective area and the rate of expected background events. We will use instrument response functions computed during the first CTA MC production (prod1) using the DESY analysis package [48]. A detailed description of the procedure used to compute these instrument response functions can be found in [49].

**Background rate**

The expected background rates for the three chosen sub-arrays are plotted in Figure 3.1. These histograms represent for each energy bin the registered rate of background events in the signal ON region after the cuts are applied. These background events are protons and electrons that survive gamma-hadron discrimination and are reconstructed as $\gamma$-ray events. The steep cosmic ray spectra introduce an energy dependence in the rate of expected background events $R_{bg}(E)$ which, as a first approximation, has a power-law shape. Efficiency of gamma-hadron separation algorithms also varies with energy. The number of background events $N_{bg}$, recorded above a given energy $E_0$, in a time $T$, is then $N_{bg} = R_{bg}T$ with $R_{bg}$ resulting from the sum of all bins in the histogram starting from the bin containing $E_0$:

$$R_{bg} = \sum_{E_i \geq E_0} R_{bg}(E_i)$$ (3.3)
Figure 3.1: Background rate as a function of energy for the three sub-arrays considered: large sub-array (red), medium sub-array (blue), small sub-array (black). These events are CR mis-reconstructed as γ-ray; the x-axis display the energy of the reconstructed γ-ray events, not the energy of the CR.

Effective area

Effective areas for the detection of γ-rays for these sub-arrays are plotted in Figure 3.2. The effective area $A_{\text{eff}}(E)$ is defined such that the differential detection rate $R_\gamma(E)$ can be written as $R_\gamma(E) = A_{\text{eff}}(E)\phi_{\text{diff}}(E)$ with $\phi_{\text{diff}}(E)$ being the differential flux of incoming γ-rays [2]. Effective areas are computed through the use of MC simulations as the ratio between the number of events recorded by the instrument and passing the background rejection algorithm $N_{\text{rec}}(E)$ and the number of initially simulated incoming γ-rays in the same energy bin, $N_{\text{MC}}(E)$:

$$A_{\text{eff}}(E) = A_0 \frac{N_{\text{rec}}(E)}{N_{\text{MC}}(E)}$$

$A_0$ is the area covered by the distribution of simulated incoming photons. For ground-based instruments, effective area is small at low energies where the Cherenkov yield of showers is not sufficient to trigger the telescope. At high energies, the effective area saturates and varies only slightly with energy as every shower within a certain distance, given by the finite FoV of the telescope, is recorded.

Since effective area depends on the energy to compute the actual surface
$A_{\text{tot}}$ over which the simulated integral fluxes are recorded, a certain shape for the spectra of the simulated sources must be assumed. We will assume that the differential flux of all the simulated sources has a power law form $\phi_{\text{diff}}(E) \propto E^{-\Gamma}$ with photon index $\Gamma = 3$, which is a typical shape for AGN VHE spectra. $A_{\text{tot}}$ can be computed as the weighted average of the effective area at all energies above $E_0$, with weights represented by the differential flux $\phi_{\text{diff}}(E)$ at the same energies:

$$A_{\text{tot}} = \frac{\sum_{E_i \geq E_0} A_{\text{eff}}(E_i) \cdot E_i^{-\Gamma} \Delta E_i}{\sum_{E_i \geq E_0} E_i^{-\Gamma} \Delta E_i}$$  \hspace{1cm} (3.5)$$

Again, the sum runs through all the bins starting from the one containing $E_0$. $\Delta E_i$ represent the width in energy of the $i$th bin of the histogram. Note that the histograms have logarithmic bins on the energy axis, therefore $\Delta E_i$ is not constant.

![Effective Area](image)

**Figure 3.2:** Effective area as a function of energy for the three sub-arrays considered: large sub-array (red), medium sub-array (blue), small sub-array (black).

The simulated light curves are intended to represent the evolution of the source flux above 100 GeV, therefore $E_0=100$ GeV. The values for the integrated effective area $A_{\text{tot}}$ and integrated background rate $R_{\text{bg}}$ computed with Eq. 3.5 and 3.3 are presented in Table 3.1.
Table 3.1: Integrated effective area and background rates for the three sub-arrays are shown. The integration extends from $E_0 = 100$ GeV to infinity. To compute $A_{tot}$, a power law source spectrum with index 3 has been assumed for the simulated sources. The instrument response functions have been optimized for 50 hours of observing time.

<table>
<thead>
<tr>
<th>Sub-array</th>
<th>$A_{tot}$ ($10^8$cm$^2$)</th>
<th>$R_{bg}$(Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>large</td>
<td>5.0</td>
<td>0.005</td>
</tr>
<tr>
<td>medium</td>
<td>2.2</td>
<td>0.003</td>
</tr>
<tr>
<td>small</td>
<td>1.6</td>
<td>0.004</td>
</tr>
</tbody>
</table>

It has to be noted that background rejection cuts optimization is different for different values of the observing time. As a consequence both the effective area and background rate depends on the observing time. When the observing time is short, loose cuts must be used in order to be able to record enough events. In CTA MC prod1 the only instruments response functions available for these sub-arrays are optimized for 50 hours of observing time.

### 3.2 Quantifying observing strategies performances

To quantify the performances of the observing strategies, the number of signal and background events within the observing windows must be computed. With these quantities, the significance of the observation can be computed as a function of time, allowing for an estimate of the probability of detecting the source. The number of signal and background events in each observing window will also be used to assesse the capability of the observing strategies to conduct successful monitoring campaigns. A C++ program has been written in order to calculate the time evolution of the number of signal and background events for each of the considered observing strategies and light curve types. This program make use of a small C++ library that has been developed for this thesis in order to provide a flexible, easy to upgrade tool for observing strategy optimization.

#### 3.2.1 Simulation loop

A series of nested loops is performed in order to explore the observing strategies parameter space. First, each combination of pointing scheme and sub-array is evaluated. Then, for a given pointing scheme and sub-array, $T_{obs}$ is varied between a minimum and a maximum observing time, $T_{obs}^{\text{min}}$ and
respectively and, for each value of $T_{\text{obs}}$, the window length takes values between $T_{\text{win}}^{\text{min}}$ and $T_{\text{win}}^{\text{max}}$.

For each simulated observing strategy, a loop over all the light curves in each set is performed. The sequence of observing windows is the same for all the light curves in the set; it is created for the first light curve and then applied to all the others. Making use of the integrated instrument response functions, $A_{\text{tot}}$ and $R_{bg}$, the number of detected signal and background events are computed as a function of time inside each window. The number $N_{\gamma}(w, t_j)$ of $\gamma$-ray events after a time $t_j$ from the beginning of the $w$th window is given by:

$$N_{\gamma}(w, t_j) = \sum_{t_l=t_s(w)}^{t_j} \phi_{LC}(t_l) A_{\text{tot}} \Delta t$$

(3.6)

where $\Delta t$ is the time interval that separates light curves points, $\Delta t = 1/\nu$ with $\nu = 1$ minute$^{-1}$ is the frequency of the light curve. The number of background events in the signal region can be estimated by multiplying the expected background rates above $E_0 = 100$ GeV with the time elapsed since the start of the window:

$$N_{bg}(w, t_j) = \sum_{t_l=t_s(w)}^{t_j} R_{bg} \Delta t$$

(3.7)

The number of signal and background events at a time $t_i$ from the beginning of the first window can be obtained by summing the contribution from all the windows before that point. Let $w'$ be the index that identifies the window containing $t_i$, $t_i \in [t_s(w'), t_e(w')]$, then:

$$N_{\gamma}(t_i) = \sum_{w=0}^{w'-1} N_{\gamma}(w) + N_{\gamma}(w', t_i)$$

(3.8)

with $N_{\gamma}(w) = N_{\gamma}(w, t_e(w))$, the total number of gamma events recorded in the $w$th window. Similarly, setting $N_{bg}(w) = N_{bg}(w, t_e(w))$, the number of background events at the instant $t_i$ from the beginning of the observation is then given by:

$$N_{bg}(t_i) = \sum_{w=0}^{w'-1} N_{bg}(w) + N_{bg}(w', t_i)$$

(3.9)

The time dependence of the background rate and the effective area is neglected in all our computations. Instruments response functions optimized for 50 hours will be used for all of the three sub-arrays.
The statistical significance of the gamma-ray excess in the signal region is computed using Equation 17 from Li and Ma [50]:

\[ S = \sqrt{2} \left\{ N_{on} \ln \left[ \frac{1 + \alpha}{\alpha} \left( \frac{N_{on}}{N_{on} + N_{off}} \right) \right] + N_{off} \ln \left[ (1 + \alpha) \left( \frac{N_{off}}{N_{on} + N_{off}} \right) \right] \right\}^{1/2} \]  

(3.10)

The factor \( \alpha \) takes into account all the differences between the observations of the ON and OFF regions. If the ON and OFF regions are observed simultaneously \( \alpha \) will be the ratio between the areas of the chosen ON and OFF region. In CTA analyses the signal-free OFF region is assumed to be five times larger than the signal (ON) region, therefore \( \alpha = 0.2 \). The number of background events in the entire OFF region is therefore \( N_{off} = N_{bg}/\alpha \). The total number of events in the ON region is the sum of the signal and background events: \( N_{on} = N_{\gamma} + N_{bg} \). Using the time dependent quantities introduced in equations from 3.6 to 3.9, the significance \( S(t_i) \) at the time \( t_i \) from the beginning of the observation can be computed as well as \( S(w) \), the significance of the detection inside the \( w \)th window.

To claim for the detection of a source, a significance greater than 5 and more than 10 \( \gamma \)-ray events are required. At each time step the condition

\[ S(t_i) > 5 \quad \land \quad N_{\gamma}(t_i) > 10 \]  

(3.11)

is tested. If this condition is satisfied for a point on the light curve, then the light curve is flagged as detected and the loop starts with the next light curve.

This process associates to each observing strategy a set of quantities that can be used to describe the performance of the strategy in detecting a variable source with given light curve’s characteristics. For the \( i \)th light curve the average flux \( \{\bar{\phi}_{LC}(w)\}_i \), the number of events in the ON and OFF region \( \{N_{on}(w)\}_i \) and \( \{N_{off}(w)\}_i \), and the significance \( \{S(w)\}_i \) are computed for each window. The time in which the condition 3.11 is satisfied, \( t_i^{det} \), and the value (0 or 1) of the detection flag \( f_i^{det} \) are also computed. Values of these quantities for all the \( N_{LC} \) light curves in each set are calculated and stored.

Observing Strategy \( \rightarrow \)

\[
\begin{align*}
&\{t_1^{det}, \ldots, t_i^{det}, \ldots, t_N^{det}\} \\
&\{f_1^{det}, \ldots, f_i^{det}, \ldots, f_N^{det}\} \\
&\{\bar{\phi}_{LC}(w)_1, \ldots, \bar{\phi}_{LC}(w)_i, \ldots, \bar{\phi}_{LC}(w)_N\} \\
&\{N_{on}(w)_1, \ldots, N_{on}(w)_i, \ldots, N_{on}(w)_N\} \\
&\{N_{off}(w)_1, \ldots, N_{off}(w)_i, \ldots, N_{off}(w)_N\} \\
&\{S(w)_1, \ldots, S(w)_i, \ldots, S(w)_N\}
\end{align*}
\]
Chapter 3.

3.2.2 Probability of detection

For each observing strategy, the measured number of detected light curves $N_{\text{det}}$ can be regarded as the number of successes obtained from a series of $N_{\text{LC}}$ identical and independent yes/no tests; it will therefore follow a binomial distribution:

$$B(N_{\text{det}}, p) = \binom{N_{\text{LC}}}{N_{\text{det}}} p^{N_{\text{det}}} (1 - p)^{N_{\text{LC}} - N_{\text{det}}}$$

(3.12)

where $p$ is the probability of success of these experiments, i.e. the probability of detecting the source with that particular strategy. Upon measuring a particular value of $N_{\text{det}}$:

$$N_{\text{det}}^{\text{meas}} = \sum_{i=1}^{N_{\text{LC}}} f_i^{\text{det}}$$

(3.13)

the best estimate of the detection probability is:

$$p_{\text{meas}} = \frac{N_{\text{det}}^{\text{meas}}}{N_{\text{LC}}}$$

(3.14)

Statistical uncertainties on the measured value of the detection probability can be expressed through the use of confidence intervals at a given confidence level $\alpha$. The method proposed by Feldman and Cousins [51], [52] has been used to construct confidence intervals that are always physically acceptable, i.e. that never violate the condition $0 \leq p \leq 1$. This method is based on Neyman’s confidence belt construction [53], [54]. For each possible value of $p$ an acceptance interval is defined as a set of values of $N_{\text{det}}$ included in $[N_{\text{det}1}(p), N_{\text{det}2}(p)]$ such that a fraction $\alpha$ of the total probability is contained inside $[N_{\text{det}1}(p), N_{\text{det}2}(p)]$, i.e. such that:

$$P(N_{\text{det}} \in [N_{\text{det}1}(p), N_{\text{det}2}(p)] | p) = \alpha$$

(3.15)

The union of all acceptance intervals constitute the confidence belt, see Figure 3.3. Upon measuring a certain value $N_{\text{det}}^{\text{meas}}$ of detected light curves Eq. 3.14 gives the best estimate of the detection probability. The corresponding confidence interval can be obtained from the confidence belt as the union of all $p$ values for which $N_{\text{det}}^{\text{meas}}$ is inside $[N_{\text{det}1}(p), N_{\text{det}2}(p)]$; for the binomial distribution this is a simply connected interval $[p_{\text{min}}, p_{\text{max}}]$.

It has to be noted that Eq. 3.15 does not define the acceptance intervals completely. A coverage condition must also be given. Following Feldman and Cousins, the acceptance interval for a given $p$ is constructed by adding values of $N_{\text{det}}$ in decreasing order of the likelihood ratio:

$$R(N_{\text{det}}, p) = \frac{B(p, N_{\text{det}})}{B(p_{\text{best}}, N_{\text{det}})}$$

(3.16)
Figure 3.3: 3σ confidence belt for the binomial distribution with $N_{\text{LC}} = 1000$. 100 values of the probability are considered.

$R(N_{\text{det}}, p)$ is the ratio of the likelihood of measuring $N_{\text{det}}$ detected light curves given a probability of $p$ over the likelihood of detecting the same number of detected light curves if the probability were $p_{\text{best}} = N_{\text{det}}/N_{\text{LC}}$, the value that maximizes $B(p, N_{\text{det}})$. Points are added until the probability inside the interval equals or exceeds $\alpha$.

Since the confidence intervals at a given confidence level depend only on the total number of light curves considered and since this number is the same for all sets, they need to be computed only once. A C program has been used to construct a look-up table containing, for every allowed value of the number of detected light curves, $N_{\text{meas}} \in [0, N_{\text{LC}}]$, values for $p_{\text{best}}$, $p_{\text{max}}$ and $p_{\text{min}}$ at a given confidence level. Confidence intervals at the 68%, 95% and 99% confidence level have been computed with a resolution of 0.0001.

3.2.3 Observing strategies parameter space

As seen in Section 3.1 an observing strategy can be defined by four elements, the instrument used and the three parameters that specify the pointing scheme. The parameter space defining the observing strategy can thus become quite large. For this reason both observing modes and sub-arrays are restricted to a few representative cases. What remains to be discussed are the ranges for the total observing time and window duration.
The choice of the range for the observing time cannot be done without considering the instrument used. In fact, as far as the achievable significance is concerned, the only difference between a large instrument and a small one is the amount of time they require to reach a certain significance level. In order to have some hint of an appropriate range for the observing time, one can refer to the much simpler case of a steady source. The significance of the observation of a constant source is found to increase as the square root of the observing time $t_{\text{obs}}$:

$$S(t_{\text{obs}}) = K \sqrt{t_{\text{obs}}}$$  \hspace{1cm} (3.17)

with $K$ being a function of the instrument characteristics, the integral effective area and background rate, and of the $\gamma$-ray integral flux of the source [30]. If the source emission is not steady then $S(t_{\text{obs}})$ will fluctuate around the value predicted by Eq. 3.17. This relation can be used to select a range for the observing times such that the exploration of the parameter space will be focused in the region in which the probability of detection changes from almost zero to almost unity. This region is the most interesting in the context of observing strategies optimization since a good observing strategy must yield almost 100% probability of success while requiring as little time as possible.

For each sub-array considered and each value of the the mean flux of the simulated sources, a plot of the significance as a function of the observing time is constructed using as input a constant light curve $\phi(t_i) = \phi_{\text{LC}} \forall t_i$ and a continuous observing mode with just one window extending from 0 to $t_{\text{obs}}$. The results are plotted in Figure 3.4 for the case of a mean light curve flux of 1% of the Crab.

For each instrument and mean flux level, the range of values for $T_{\text{obs}}$ is constructed requiring that:

$$T_{\text{obs}}^{\text{min}} \mid S(T_{\text{obs}}^{\text{min}}) = 2 \hspace{1cm} T_{\text{obs}}^{\text{max}} \mid S(T_{\text{obs}}^{\text{max}}) = 8$$  \hspace{1cm} (3.18)

The chosen values for the extremes of the interesting significance interval, 2 and 8 respectively, have been matched to the size of the fluctuations introduced by sources’ variability. Fitting the plots with a function of the form of 3.17, the conditions 3.18 can be solved analytically to compute the extremes of the range for the observing time. The results are presented in Table 3.2. Since the agreement between the data points and the fit is very good, relative uncertainties on the extremes of the observing time range are of the order of $\sim 10^{-6}$. It has to be noted that the use of instrument response functions optimized for 50 hours of observation will result in some cases in an underestimation of sub-arrays’ performances. This happens when the explored $T_{\text{obs}}$ range is far from the optimization time (50h). The large sub-array will be particularly affected, especially for sources with 1% Crab mean flux.
Figure 3.4: Significance as a function of the observing time for the three sub-array considered: large 23 MSTs sub-array (red), medium sized 9 MSTs sub-array (green), small 4 MSTs sub-array (blue)

<table>
<thead>
<tr>
<th>Sub-array</th>
<th>$\bar{\phi}_{LC} = 1%$ Crab</th>
<th>$\bar{\phi}_{LC} = 0.5%$ Crab</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>[0.4, 7.2] h</td>
<td>[1.6, 25] h</td>
</tr>
<tr>
<td>Medium</td>
<td>[1.3, 21] h</td>
<td>[5, 78] h</td>
</tr>
</tbody>
</table>

Table 3.2: Ranges for the total observing time for light curves with mean fluxes of 1% and 0.5% of the Crab.

The choice of the minimum window length is influenced by the slewing capability of the MSTs. Estimating that one minute is needed to repoint the telescopes to any location in the sky, values of $T_{win}^{min}$ around the minute can be discarded for they will result in a loss of a considerable fraction of the live time of the instrument in pointing operations. The choice has been to set $T_{win}^{min}$ to ten minutes, corresponding to the loss of $\sim10\%$ of the observing time in telescopes’ pointings. The maximum value for the observing window length, $T_{win}^{max}$, has been set to

$$T_{win}^{max} = \min\{12\ h, \ T_{obs}^{max}\}$$  \hspace{1cm} (3.19)$$

Except for observations of sources with 1% Crab mean flux with the large sub-array, the maximum window length will always be of twelve hours repre-
senting an observation carried through a whole night. This is an ideal case: in reality no source is visible for more than two to four hours per night. For the periodic observing mode, twelve hours is also the distance between the starting points of the windows. A periodic observing mode with a maximum window length results in a continuous observation of the light curve.
Chapter 4

Selection of the best observing strategies

The previous chapter describes the ways in which the observing strategies are simulated and how their performances are quantified. In this chapter we show how this information can be used to select the strategy that is best suited to carry out a given type of observation. Two different scenarios are considered:

- the detection of a source in as little time as possible
- monitoring a variable source.

The goal of the observation defines the requirements on what is considered the best strategy; different definitions of what is “best” must be used in each of these cases. The best strategy ensures a good compromise between performances, here evaluated in terms of the probability of detection, and the usage of resources, time and number of telescopes. For each of the two considered scenarios, a hierarchy among these parameters is built, reflecting the importance each one has in defining what is the best strategy. The best strategy is chosen by ordering the observations according to this hierarchy of parameters.

In Section 4.1 we show how the detection probability depends on the parameter defining the observing strategy. In Sections 4.2 and 4.3 the method used to select the best observing strategy for each of the two considered scenarios, fastest detection and source monitoring, is described and the results are presented. The impact of the simulated light curve properties (see Section 2.3.3) on the observing strategies’ optimization is discussed in Section 4.4. Finally, in Section 4.5 we apply our analysis to the specific case of monitoring the BL Lac Mrk 421 with VERITAS.
4.1 Observing strategies performances

The key parameter we use to evaluate the performance of an observing strategy is the probability of detecting the source, \( p_{\text{det}} \). The analysis presented in the previous chapter allows us to inspect how \( p_{\text{det}} \) is affected by the parameters that define an observing strategy: the sub-array type, the observing mode (random or periodic), the total observing time \( T_{\text{obs}} \) and the window duration \( T_{\text{win}} \).

The results presented here are obtained from the analysis of the reference light curves, defined in Section 2.3.3 and characterized by a mean flux above 100 GeV of 1\% of the Crab nebula, a 1.15 PSD index, 50\% fractional variance and 50\% non-linearity index.

4.1.1 Detection probability in the \((T_{\text{win}}, T_{\text{obs}})\) plane

For each sub-array and observing mode, the evolution of \( p_{\text{det}} \) in the \((T_{\text{win}}, T_{\text{obs}})\) plane is presented in Figure 4.1.

Two empty regions are visible, caused by conflicting requests in the exploration of the parameter space. The first one appears in the bottom-right part of the \((T_{\text{win}}, T_{\text{obs}})\) plane, in the region \( T_{\text{obs}} < T_{\text{win}} \). This empty region is caused by the fact that the condition \( T_{\text{win}} < T_{\text{obs}} \) is not satisfied for all the simulated observing strategies, as \( T_{\text{min}}^{\text{obs}} \) is always shorter than \( T_{\text{max}}^{\text{win}} \). For the case of the periodic observations, another empty region appears on the left, near the \( y \)-axis. For this observing mode, the maximum number of windows is limited by the finite length of the simulated light curves. For the 500h-long light curves used, only 41 nights are fully available for placing the observing windows. Small values of the window length \( T_{\text{win}} \), for which \( 41 \cdot T_{\text{win}} < T_{\text{obs}} \), results in the impossibility of covering all the observing time. This empty region is particularly noticeable for the small and medium sub-arrays, due to the larger values of \( T_{\text{obs}} \) that are considered in these cases (see Section 3.2.3).

Looking at Figure 4.1 some important conclusions can be drawn. As might be expected, for a fixed value of the window length \( T_{\text{win}} \), the detection probability increases with the observing time. However, the gradient along the \( y \)-axis is stronger for small values of the window length. This means that a 100\% probability of detection can be achieved in a smaller amount of time if this is split in many small windows compared to the case in which only a few, large windows, are employed. This is particularly evident when the periodic observing mode is used.

A probable explanation for the strong dependence of \( p_{\text{det}} \) on the duration of the observing windows is that the use of many small windows increases the chances of observing, within one window, a strong, positive fluctuation...
Figure 4.1: Two dimensional histograms of the detection probability for the reference light curves as a function of $T_{\text{obs}}$, $T_{\text{win}}$ for the three sub-arrays and the two observing modes. $T_{\text{obs}}$ and $T_{\text{win}}$ are expressed in seconds, the probability is shown in the color scale. From top to bottom: large sub-array, medium sub-array and small sub-array. Histograms on the left side are relative to the periodic observing mode, while the ones on the right are relative to the random one.
in the source’s flux. Such a window will then have a high signal-to-noise ratio and, as a consequence, will increase substantially the significance of the detection.

### 4.1.2 Detection probability as a function of the observing time

To study how different observing modes influence the behavior of the detection probability, for each sub-array and observing mode the maximum detection probability is represented as a function of the observing time $T_{\text{obs}}$.

To create these plots, each observing time is assigned the maximum value of the detection probability that can be found among the strategies that use that observing time. Referring to Figure 4.1, at each observing time $\tilde{T}_{\text{obs}}$ we attribute the maximum value of the probability that can be found moving along the horizontal line of equation $T_{\text{obs}} = \tilde{T}_{\text{obs}}$. The results for the three considered sub-arrays are shown in Figure 4.2.

In all cases these plots show a smooth rise in detection probability as the total observing time increases. Apart from the case of the medium sub-array, we see that the observing mode does not influence the maximum detection probability that can be achieved in a given amount of time. The two curves are always compatible at a $3\sigma$ confidence level for every value of $T_{\text{obs}}$. In the case of the medium sub-array, the two observing modes yield compatible results for observing times less than $\sim10h$. For larger values of the observing time, the random pointing scheme performs better than the periodic one, reaching 100% detection probability roughly 2 hours earlier than the periodic observing mode.

The result for the small sub-array can be used to test the reliability of this analysis. This sub-array of 4 MSTs is similar, in telescopes design and position on the ground, to VERITAS (see Section 1.4) and their performances are comparable. As we can see from the bottom panel of Figure 4.2, the small sub-array is able to detect a variable source with a 1% Crab mean flux in roughly 25h. This is the same amount of time required by VERITAS for detecting sources of comparable strength.

**Comparison with a continuous observation**

In order to evaluate the importance of spreading the available observing time in small windows, also a continuous, uninterrupted, observation of the source has been simulated. This continuous observing mode makes use of, for every considered value of $T_{\text{obs}}$, a unique observing window that extends from 0 to
Figure 4.2: Maximum detection probability as a function of time for each sub-array and observing mode. Only 25 values for $T_{\text{obs}}$ are shown. Blue circles refer to the periodic observing mode while red squares refer to the random one. The errors show the $3\sigma$ confidence intervals (see Section 3.2.2). Top: Large sub-array. Middle: medium sub-array. Bottom: small sub-array.
It has to be noted that this observing mode is not realistic, as night-day alternation prevent truly continuous observations.

In Figure 4.3, \( p_{\text{det}}(T_{\text{obs}}) \) is presented for the two observing modes, random and periodic, that use windows together with the results for the continuous observing mode. The same procedure and set of reference light curves used to produce the plots for the random and periodic observing modes has been used for the analysis of this third pointing scheme.

![Figure 4.3: Maximum detection probability as a function of the observing time for the small sub-array for the random (red squares), periodic (blue filled circles), and continuous observing mode (black triangles).](image)

The two curves are compatible for \( T_{\text{obs}} \lesssim 17 \) hours; however, for longer observations the probability achieved with the continuous observing mode is always far below the one yielded by the two observing modes that split the observing time in windows. While the strategies that split the observing time in windows achieve 100% probability of detection in \( \sim 25 \) h, the continuous observation does not even reach 100% in the considered \( T_{\text{obs}} \) range. This confirms that, as a consequence of fluctuations in the light curves, observing modes that divide the observing time in windows allows for a faster detection of the source.

As an additional test, we have repeated the analysis using a set of white-noise light curves as input. White-noise light curves are linear light curves with a flat PSD (\( \beta = 0 \)); light curves of this type will therefore lack the fluctuations that can be responsible for the dependence of \( p_{\text{det}} \) on both window
length and observing mode. In Figure 4.4, we compare the $p_{det}(T_{obs})$ plots relative to the analysis of the set of white-noise light curves with the ones obtained for the reference light curves. Only the small sub-array is shown; the same conclusions apply to all the other three sub-arrays.

![Figure 4.4: Maximum detection probability as a function of the observing time for the small sub-array as computed with a set of white noise light curves. Black curves refer to random (squares) and periodic (filled circles) observing mode, the red curve with triangles is the result of a continuous observation.](image)

No noticeable difference is present between the pointing schemes that employ many windows and the continuous one. For all the three observing modes, $p_{det}$ shows a steep rise, passing from 0 to 1 in roughly two hours. In fact, for white noise light curves the mean of light curves segments is constant along the light curve. The situation is then similar to the case of a perfectly constant light curve, where $p_{det}$ is expected to be a step function.

### 4.2 Best strategy to detect a source in as little time as possible

At the first level, the best observing strategy for each sub-array and observing mode must be selected with respect to $T_{obs}$ and $T_{win}$. We define the best strategy as the one which satisfies:

i) The probability of detection is maximum.
ii) The observing time is minimum.

iii) The window length is maximum.

The last request is motivated by the desire to minimize the time loss in repointing the telescopes. Strategies that concentrate the observing time in few large windows are then preferred to ones in which the observations are spread out along the light curve. These three criteria are evaluated in hierarchical order: first the observing strategies are sorted according to the value of $p_{\text{det}}$ and the maximum detection probability, $p_{\text{det}}^{\text{max}}$, that can be achieved with that sub-array and observing mode is found. A subset is then extracted containing all the strategies whose detection probability is compatible at a three sigma confidence level with $p_{\text{det}}^{\text{max}}$. The construction used to choose the intervals for $T_{\text{obs}}$, described in Section 3.2.3, results in $p_{\text{det}}^{\text{max}} = 1$ for all the considered cases. This subset of observing strategies for which the detection probability is maximum is sorted by $T_{\text{obs}}$. In the case of strategies with the same value of the observing time, the one with the longest window is chosen. At the end of this sorting procedure, the best strategy, according to the definition given above, can be found for each combination of sub-arrays and observing modes. Referring to the histograms in Figure 4.1 the best strategy, located at the point $(T_{\text{win}}^{\text{best}}, T_{\text{obs}}^{\text{best}})$, is obtained moving as far down as possible along the $y$-axis ($T_{\text{obs}}$) and as far right as possible along the $x$-axis ($T_{\text{win}}$) without overstepping the maximum probability region.

At the second level, the best strategy to operate each sub-array is selected. To make this choice, the two observing modes are compared and the best among them is selected according to the definition of best given above. Since after the first step of the selection $p_{\text{det}}$ is now always compatible with 100% for both observing modes, the best one to operate each sub-array is chosen by looking at the values for $T_{\text{obs}}^{\text{best}}$ and $T_{\text{win}}^{\text{best}}$. The best strategy will be the one that has the smaller $T_{\text{obs}}^{\text{best}}$. If the values of $T_{\text{obs}}^{\text{best}}$ for the two observing modes are the same, within the precision of the simulation, then the strategy that uses larger observing windows will be preferred.

At the third and final level, the overall best strategy that ensures the fastest detection of a source can be found. The large array, with larger effective area, is faster in terms of the total observing time. However, the number of telescopes used must also be taken in consideration. Being composed of 23 telescopes, the large sub-array needs roughly the same resources as 2.6 medium-sized sub-arrays of 9 telescopes and 5.75 small 4 telescope sub-arrays. As a consequence it can be said that while the large instrument observes a single source, 2.6 and 5.75 “sources” can be observed simultaneously using the medium and small sub-arrays respectively. To take this fact into account upon comparing different instruments their values of $T_{\text{obs}}^{\text{best}}$ must
be multiplied by the number of telescopes $N_{tel}$ composing each instrument. The overall best strategy will be the one which yields the minimum value of this product.

The results of this procedure, for the analysis of the reference light curves, are presented in Tables 4.1 and 4.2. Table 4.1 shows the values of $(T_{win}^{best}, T_{obs}^{best})$ corresponding to the best strategies for each sub-array and observing mode. Since the procedure used to select the best strategy results in the identification of one single point in each histogram, quoted errors on $T_{win}$ and $T_{obs}$ are simply expressing the width of the time bins used. Table 4.2 shows the values of the $T_{obs}^{best} \cdot N_{tel}$ products for the best strategies to operate each sub-array.

<table>
<thead>
<tr>
<th>Sub-array</th>
<th>Random</th>
<th>Periodic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{win}^{best}$ (10$^2$s)</td>
<td>$T_{obs}^{best}$ (10$^4$s)</td>
</tr>
<tr>
<td>Small</td>
<td>6 ± 4</td>
<td>8.3 ± 0.2</td>
</tr>
<tr>
<td>Medium</td>
<td>6 ± 4</td>
<td>4.24 ± 0.07</td>
</tr>
<tr>
<td>Large</td>
<td>6 ± 3</td>
<td>1.66 ± 0.02</td>
</tr>
</tbody>
</table>

**Table 4.1:** Values for $(T_{win}^{best}, T_{obs}^{best})$ of the best strategies for each sub-array and observing mode in the case of fastest detection as computed for the reference light curves.

<table>
<thead>
<tr>
<th>Sub-array</th>
<th>$T_{obs}^{best}$ · $N_{tel}$ (10$^5$s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small sub-array</td>
<td>(3.3 ± 0.6)</td>
</tr>
<tr>
<td>Medium sub-array</td>
<td>(3.8 ± 0.6)</td>
</tr>
<tr>
<td>Large sub-array</td>
<td>(3.8 ± 0.5)</td>
</tr>
</tbody>
</table>

**Table 4.2:** Values for the product $T_{obs}^{best} \cdot N_{tel}$ for the best strategies achievable with all the three different sub-arrays. The units are 10$^5$s.

By looking at Table 4.1, we see that a small value of $T_{win}$ is preferred in all cases. This was already evident from the histograms in Figure 4.1. For the random observing mode, the fastest detection is achieved in every case by strategies that uses the smallest considered value for $T_{win}$. For the periodic observing mode, with the small and medium subarrays, larger windows must be used, as the finite length of the light curves prevents detection with small windows. By comparing the values of $T_{obs}^{best}$ relative to the two observing modes, we see that the best strategy to operate all three sub-arrays, when a fast detection of a source with properties of the reference light curve is desired, always uses the random observing mode.

The selection of the overall best strategy can be done according to the value of the $T_{obs}^{best} \cdot N_{tel}$ product, reported in Table 4.2. According to these
values, the simultaneous use of small sub-arrays operated with a random observing mode and 10 minutes windows is the best choice when fast detection of sources is desired. Performances of the medium and large sub-arrays are comparable with each other. However, it has to be mentioned that the performances of the large sub-array are slightly underestimated due to the use of instrument response functions computed for 50h when the range of $T_{obs}$ does not exceed 8 h. We must conclude therefore that the three sub-arrays considered have almost comparable performances. Other criteria can then be used to decide what sub-arrays to use. A Larger effective area and a better angular and energy resolutions naturally favor the use of the large sub-array.

### 4.3 Best AGN monitoring strategy

Monitoring campaigns are done in order to gain information about the long term behavior of sources. The goal of a monitoring campaign is to “cover” the source with many observations spread over long periods of time. Each observing window should be considered as an independent observation. Since the aim is to detect the source in each window, the sensitivity achievable in a monitoring campaign is limited by the fact that the observing time must fit within one night. For this reason, current IACT instruments can only monitor the brightest AGN. CTA, with its greatly increased effective area, will be able to monitor a larger number of sources, as many more weak AGN will be within reach of the instrument.

The total observing time, $T_{obs}$, is not a relevant parameter when looking for the best monitoring strategy. While each window represents an independent observation, there is no difference between the random and the periodic observing mode. We will therefore limit our analysis to the periodic observing mode only. In the simulation, the window length will be varied from zero to twelve hours, the latter representing the ideal case of a full night dedicated to the monitoring of a single source\(^\text{1}\). For each value of $T_{win}$, the total observing time will be defined by the request of having an integer number of windows: $T_{obs} = N_{win} T_{win}$. The presence of smaller windows, resized to fit the total observing time, is then avoided. All the windows will have exactly the same duration. To investigate CTA monitoring capabilities, we will use four different sets of light curves, each characterized by a different mean, but with all other light curve parameters set at their reference value (see Section 2.3.3). The considered values of the source’s mean flux are: 1%, 3%, 10%, and 30% of the Crab.

\(^{1}\)It should be noted that this is an ideal and unrealistic case since no source is visible for the whole night.
Chapter 4.

The performances of the monitoring strategies will be evaluated through the use of the detection probability in each window. What concerns the choice of the best monitoring campaign is to find the smallest window length that ensures a high detection probability in a large fraction of the windows. The best monitoring strategy to operate each sub-array can be then defined as a strategy for which:

i) \( P_{\text{det}} > 95\% \) in at least 90\% of the windows

ii) The window length is minimum.

The probability of detecting the source inside each window is evaluated with same procedure as described in Section 3.2.2, starting from the information on the window-wise significance and number of \( \gamma \)-ray events in each window (Section 3.2). A small value of \( T_{\text{win}} \) is important as the interference of the monitoring campaign with other observations will be minimized. Small windows also implies that many sources can be monitored in the same night.

Again, these two conditions are evaluated in hierarchical order. First, from among all the observing strategies a subset is drawn containing all the strategies for which more than 90\% of the windows have a detection probability \( > 95\% \). Among these strategies the best one is the one with smallest window length. In this way the best monitoring strategy to operate each sub-array can be found. The overall best monitoring strategy will be the one that maximizes the number of sources that can be monitored within one night (12h, 4.32 \( \cdot \) 10\(^4\)s). Considering that, for each window, an additional minute will be lost in slewing the telescopes from one source to the other, the number of sources that can be monitored with one sub-array can be computed as:

\[
N'_{\text{mon}} = \frac{4.32 \cdot 10^4 \text{s}}{T_{\text{win}}^{\text{best}}(\text{s}) + 60\text{s}}
\]  

(4.1)

The number of telescopes used by each sub-array must be considered. The large sub-array is equivalent to two medium sized, nine-telescope sub-arrays or to five four-telescope small sub-arrays. While the large array can only observe a single source at the time, two and five sources can be monitored simultaneously with the medium and small sub-arrays respectively. To take this into account, the number of sources that can be monitored with one sub-array, \( N'_{\text{mon}} \), is multiplied by the number \( N_{\text{SA}} \) of sub-arrays that can operate simultaneously:

\[
N_{\text{mon}} = N'_{\text{mon}} \cdot N_{\text{SA}}
\]  

(4.2)

where \( N_{\text{SA}}^{\text{large}} = 1 \), \( N_{\text{SA}}^{\text{med}} = 2 \), and \( N_{\text{SA}}^{\text{small}} = 5 \).
Chapter 4.

Table 4.3 shows the results of the procedure used to select the best monitoring strategy for each sub-array. The errors on $T_{\text{best}}^{\text{win}}$ are given by the width of the bin used in the simulation. The number of sources, $N_{\text{mon}}$, that can be monitored within one night for the considered sub-arrays and mean fluxes is presented in Table 4.4.

<table>
<thead>
<tr>
<th>$\phi_{\text{LC}}$ (% of Crab)</th>
<th>Large sub-array</th>
<th>Medium sub-array</th>
<th>Small sub-array</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>6 ± 2</td>
<td>6 ± 2</td>
<td>6 ± 2</td>
</tr>
<tr>
<td>10%</td>
<td>10 ± 2</td>
<td>27 ± 2</td>
<td>49 ± 2</td>
</tr>
<tr>
<td>3%</td>
<td>62 ± 2</td>
<td>155 ± 2</td>
<td>296 ± 2</td>
</tr>
<tr>
<td>1%</td>
<td>364 ± 2</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

Table 4.3: Window length ($10^2$s) of the best monitoring strategies with periodic observing mode for each sub-array and value of the source’s mean flux. Errors represent the width of the bin used in the simulation.

<table>
<thead>
<tr>
<th>$\phi_{\text{LC}}$ (% of Crab)</th>
<th>Large sub-array</th>
<th>Medium sub-array</th>
<th>Small sub-array</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>$(6 ± 2) \cdot 10$</td>
<td>$(1.3 ± 0.4) \cdot 10^2$</td>
<td>$(3 ± 1) \cdot 10^2$</td>
</tr>
<tr>
<td>10%</td>
<td>41 ± 8</td>
<td>31 ± 2</td>
<td>43 ± 2</td>
</tr>
<tr>
<td>3%</td>
<td>6.9 ± 0.2</td>
<td>5.5 ± 0.1</td>
<td>7.30 ± 0.05</td>
</tr>
<tr>
<td>1%</td>
<td>1.18 ± 0.01</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

Table 4.4: Values for $N_{\text{mon}}$, the number of sources that can be simultaneously monitored within one night, for the large, medium and small sub-arrays and different values of the source’s mean flux.

From Table 4.3 we see that a strong source with a mean flux of 30% of the Crab Nebula can be detected in ten minutes by all three sub-arrays. Monitoring of weaker sources requires increasingly larger windows. A source with a 1% Crab mean flux can still, in principle, be monitored by CTA if all the MSTs are used for that purpose. However, the monitoring window must be extended for $\sim 10$ hours, which is too long to be realistic, as sources are only visible for two to four hours per night. Considering this, sources with the reference light curve characteristics and a mean flux of 1% Crab level are beyond the monitoring capabilities of CTA’s MSTs.

Taking into account the total number of sources that can be monitored within one night (Table 4.4), we conclude that for each of the considered mean flux values, extensive monitoring campaigns are best conducted by the simultaneous usage of five small four-telescope sub-arrays.
4.4 Impact of light curves’ characteristics

To inspect how the best strategy evolves with source properties and which light curves parameter affect AGN observations the most, we extend the analysis presented in Sections 4.2 and 4.3 to light curves characterized by different properties. Four parameters are needed to define the light curves: the mean flux, the fractional variance, the PSD index and the non-linearity index. In order to evaluate how each of these parameters influences the choice of the best strategy, we repeat the analysis with sets of light curves that differ from the reference light curves by the value of one of these parameters (Section 2.3.3). Another parameter that can influence the results is the assumed photon index for the source. In this work the impact of variation of the photon index on the results has not been tested.

The conclusions drawn in Section 4.2 have been obtained with the set of reference light curves, characterized by a mean flux above 100 GeV of 1% of the Crab nebula, a 1.15 PSD index, 50% fractional variance and 50% non-linearity. Light curve with a smaller mean, 0.5% of the Crab will be considered. The chosen variation of the fractional variance, 20% and 80% are representative of the highest and lowest fractional variances found in the light curves of AGN included in the Fermi LAT Second AGN Catalog. Two alternate values of the PSD index have also been used: $\beta=1.7$, measured by the Fermi Space Telescope for a sample of the brightest BL Lacs in the first LAT AGN catalog, and $\beta=2$, a value measured by HESS at energies above 200 GeV for the PKS 2155-304 light curve during the 2006 flare. Considered variations of the non-linearity of the light curves are 20% and 80%.

4.4.1 Impact of light curves’ characteristics on the detection probability

In Figures 4.5 and 4.6, we show the two dimensional histograms of the detection probability in the $(T_{\text{win}}, T_{\text{obs}})$ plane as resulting from the analysis of the different sets of light curves. These histograms refers to the medium sub-array. The variation of the detection probability with the light curve’s characteristics follow the same patterns for the other two sub-arrays.

Beside the obvious importance of the mean flux level, it appears that light curves’ fractional variance and PSD index have a strong impact on the detection probability. Variations of the non-linearity of the light curves only marginally affect the detection probability.

The variance is a measure of the spread of light curves’ points around the mean value. Light curves with small variance have their points closer to the mean values. As a consequence, the scale of the flux variations is
Figure 4.5: Two dimensional histograms of the detection probability as a function of $T_{\text{obs}}, T_{\text{win}}$ for the medium sub-array for the different sets of light curves examined. From top to bottom: reference light curves, 0.5% Crab mean flux, 80% $F_{\text{var}}$ and 20% $F_{\text{var}}$. Histograms on the left refer to the periodic observing mode while the histograms on the right side refer to the random one.
Figure 4.6: Continuation of Figure 4.5. From top to bottom: 80% non-linearity, 20% non-linearity, 1.7 PSD index and 2.0 PSD index.
reduced, and the dependence of the detection probability on the length of the windows is less accentuated. When variance is increased, both positive and negative flux fluctuations are enhanced and the detection probability is more influenced by the window duration. The PSD index controls the amount of variability power in each frequency band. Increasing the PSD index results in a transfer of power from the high to the low frequency components of the Fourier spectrum. Long term fluctuations are enhanced and the resulting light curves are characterized by larger, although more widely separated, high-flux periods. For this reason, when the PSD index increases, the impact of window duration also increases.

As we can see from Figures 4.5 and 4.6, the impact of light curve parameters on the detection probability is more marked for strategies that use periodic observing mode and large windows. The probability of detecting a source is less affected by the properties of the light curves when strategies with small windows are used.

### 4.4.2 Impact of light curves’ characteristics on the best strategy for fast detection

By applying the techniques described in 4.2, the best strategies for the different light curves types can be selected. The results are presented in Table 4.5.

<table>
<thead>
<tr>
<th>Light curve type</th>
<th>sub-array</th>
<th>pointing</th>
<th>$T_{\text{win}} (10^3 \text{s})$</th>
<th>$T_{\text{obs}} (10^4 \text{s})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>reference light curves</td>
<td>small</td>
<td>random</td>
<td>6 ± 4</td>
<td>8.3 ± 0.2</td>
</tr>
<tr>
<td>$\phi_{\text{LC}} = 0.5%\text{Crab}$</td>
<td>small</td>
<td>random</td>
<td>23 ± 4</td>
<td>28.6 ± 0.6</td>
</tr>
<tr>
<td>$F_{\text{var}} = 20%$</td>
<td>large</td>
<td>periodic</td>
<td>6 ± 3</td>
<td>1.27 ± 0.02</td>
</tr>
<tr>
<td>$F_{\text{var}} = 80%$</td>
<td>small</td>
<td>periodic</td>
<td>23 ± 4</td>
<td>9.2 ± 0.2</td>
</tr>
<tr>
<td>$\beta = 1.7$</td>
<td>small</td>
<td>random</td>
<td>6 ± 4</td>
<td>8.0 ± 0.2</td>
</tr>
<tr>
<td>$\beta = 2.0$</td>
<td>small</td>
<td>random</td>
<td>23 ± 4</td>
<td>8.0 ± 0.2</td>
</tr>
<tr>
<td>$I_{\text{lin}} = 80%$</td>
<td>small</td>
<td>random</td>
<td>6 ± 4</td>
<td>8.5 ± 0.2</td>
</tr>
<tr>
<td>$I_{\text{lin}} = 20%$</td>
<td>small</td>
<td>random</td>
<td>6 ± 4</td>
<td>8.5 ± 0.2</td>
</tr>
</tbody>
</table>

**Table 4.5:** Variations of the best strategy for a fast detection of the source with different light curves’ parameters. We remind that the reference light curves are characterized by: $\phi_{\text{LC}} = 1\%\text{Crab}, F_{\text{var}} = 50\%, \beta = 1.15, I_{\text{lin}} = 50\%$.

The time needed to detect a variable source increases with the variance of the light curves, as can be seen in Table 4.5 comparing the results for the reference light curves with the one for light curves with $F_{\text{var}} = 80\%$. On the
contrary, when PSD index increase, sources are detected in a smaller amount of time, because the fluctuations become wider. Non linearity does not have a strong influence on the choice of the best strategy. As shown in Table 4.5, for $I_{\text{nlin}} = 20\%$, $I_{\text{nlin}} = 50\%$ (reference light curves) and $I_{\text{nlin}} = 80\%$ the values of $T_{\text{best}}^{\text{obs}}$ are always compatible with each other.

With the exception of light curves that have a low fractional variance, the simultaneous use of small sub-arrays is always preferred when a fast detection of the source is desired. Even in the case of light curves with a 20% fractional variance, there is only a marginal advantage in using the large sub-array, as the difference in the value of the product $N_{\text{tel}} \cdot T_{\text{best}}^{\text{obs}}$ between the small and large sub-arrays is small compared to the errors.

The best observing strategies always use the random observing mode, except for the case of light curves with low and high fractional variances. When $F_{\text{var}} = 80\%$, for the small sub-array, both observing modes have the same value for $T_{\text{best}}^{\text{obs}}$; the periodic observing mode is preferred since it allows larger windows to be used. When $F_{\text{var}} = 20\%$, for the large sub-array, the periodic observing mode has a lower $T_{\text{best}}^{\text{obs}}$. We see that, in all cases, to detect a source in as little time as possible, small windows must be used.

4.4.3 Impact of light curves’ characteristics on the best monitoring strategies

We analyze how the light curves’ parameters influence the choice of the overall best monitoring strategy. In Table 4.6 the values of the window length for the best monitoring strategies are presented for the three sub-arrays and the different light curve sets.

As already discussed in the previous sections, a light curve’s variance has a strong impact on sources’ detectability. Sources that exhibit large values of the fractional variance are more difficult to detect, meaning longer windows are required to monitor such sources. The effect of increasing the fractional variance is so strong that only the large sub-array will be able to monitor a source with a mean flux of 3% of the Crab if light curve’s $F_{\text{var}}$ is 80%. In this case the window length required is $\sim 7$ h. Monitoring of sources of this type will then require all CTA MSTs to be used for the majority of the time, which is not feasible. It must be concluded that sources with a high 80% fractional variance and mean flux of the order of 3% of the Crab are out of the monitoring capabilities of the three sub-arrays considered.

Increasing the PSD index enhances fluctuations at longer time scales, while reducing the amount of variability at the highest frequencies: the duration of periods with high flux levels increases as well as the average spacing
between these periods. For this reason, the duration of the monitoring windows also increase with the PSD index. Again, sources with a 3% mean flux and light curves characterized by a PSD index of 2 can not be monitored with CTA’s MSTs.

Table 4.6: Variation on the window length (10^2 s) of the best monitoring strategies (periodic observing mode) for each sub-array and light curve type considered

<table>
<thead>
<tr>
<th>$\bar{\phi}_{LC}(%Crab)$</th>
<th>Light curve type</th>
<th>$T_{win}(10^2s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>reference</td>
<td>large</td>
</tr>
<tr>
<td>10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{var} = 20%$</td>
<td>10±2</td>
<td>27±2</td>
</tr>
<tr>
<td>$F_{var} = 80%$</td>
<td>6±2</td>
<td>10±2</td>
</tr>
<tr>
<td>$I_{nlin} = 20%$</td>
<td>52±2</td>
<td>121±2</td>
</tr>
<tr>
<td>$I_{nlin} = 80%$</td>
<td>19±2</td>
<td>40±2</td>
</tr>
<tr>
<td>$\beta = 1.7$</td>
<td>23±2</td>
<td>69±2</td>
</tr>
<tr>
<td>$\beta = 2.0$</td>
<td>40±2</td>
<td>100±2</td>
</tr>
<tr>
<td>3%</td>
<td>reference</td>
<td>62±2</td>
</tr>
<tr>
<td>$F_{var} = 20%$</td>
<td>27±2</td>
<td>70±2</td>
</tr>
<tr>
<td>$F_{var} = 80%$</td>
<td>253±2</td>
<td>/</td>
</tr>
<tr>
<td>$I_{nlin} = 20%$</td>
<td>83±2</td>
<td>193±2</td>
</tr>
<tr>
<td>$I_{nlin} = 80%$</td>
<td>49±2</td>
<td>125±2</td>
</tr>
<tr>
<td>$\beta = 1.7$</td>
<td>160±2</td>
<td>385±2</td>
</tr>
<tr>
<td>$\beta = 2.0$</td>
<td>291±2</td>
<td>/</td>
</tr>
</tbody>
</table>

The choice among the sub-arrays is based on the maximum number of sources that can be monitored within one night as described in Section 4.3. When the mean source flux is 10% of the Crab, the simultaneous use of small sub-arrays appears to be the best option for conducting monitoring campaigns. Weaker sources, with a 3% Crab mean, characterized by high fractional variance or PSD index of 1.7 and 2.0, are out of the monitoring capability of the considered sub-arrays. For monitoring these sources, the required observing windows are always larger than the typical 2-4 hours per night in which sources are visible.
4.5 Application to a current IACT: Monitoring of Mrk 421 with VERITAS

Although developed for CTA, this analysis can be easily applied to optimize the observing strategies of any other IACT facility, given that the instrument response functions are known.

As an example we consider the VERITAS array of IACTs. The design of the four telescopes of VERITAS are very similar to the one used for the simulation of CTA MSTs. The CTA 4 MST sub-array, is also very similar to the VERITAS configuration in terms of telescope spacing. For this simple application, VERITAS instrument response functions can be approximated with the one computed for the CTA’s small 4-MST sub-array.

We explore the specific case of the monitoring of the BL Lac object Markarian 421 (Mrk 421). In order to simulate Mrk 421’s flux variability, the values for the source mean flux, its light curves’ fractional variance, and PSD index are needed. Mrk 421 light curves detected by Fermi-LAT in the HE band shows a PSD index of 1.7 and a fractional variance of 24% [41]. The source VHE mean flux is roughly 30% of the Crab [55]. These parameters will be used to characterize the Mrk VHE light curve, assuming that the variability properties observed at HE are the same at VHE. With these parameters, and assuming a non-linearity index of 50%, Mrk 421 light curves are simulated with the algorithm described in Section 2.3.2.

Using these light curves, the best strategy for monitoring Mrk 421 with VERITAS is found according to the method presented in Section 4.3. According to this method, and considering the hypothesis made about Mrk 421’s variability characteristic, the optimal monitoring procedure would be to observe the source once per night with windows of 10±3 minutes length.

We note that Mrk 421 is an extremely variable source, with light curve properties that change significantly from one year to the next [56]. A full exploration of the light curves’ parameter space should be done, in order to find the best monitoring plan for the source, based on its unique properties. Such a study is beyond the scope of this work.
Outlook and conclusions

In this work we have developed a tool that can be used to optimize IACTs observing strategies for variable sources. This has required the simulation of AGN light curves and the creation of a toy model of the observing strategies.

The light curve simulation algorithm produces light curves with a power law PSD and a linear relationship between the RMS amplitude and the mean flux, characteristics that are observed in HE and VHE AGN light curves. The model of the observing strategies takes into account both the response of the IACTs and the pointing scheme used to carry on the observations. Three sub-arrays of MSTs have been considered: a 23-telescope large sub-array, a 9-telescope medium sized sub-array and a 4-telescope small sub-array. The instrument response functions used to represent these sub-arrays are from the first CTA MC production. Two pointing schemes are implemented, in which a random and a periodic prescription is used for placing the observing windows.

This tool can be used to find the optimal way to operate CTA. As an illustration, two observing goals have been considered in this work: fast detection and source monitoring. Sets of reference light curves, with a PSD index of 1.15, a mean flux of 1% of the Crab, a 50% fractional variance and a 50% non-linearity index have been used. For monitoring strategies, other values for the mean flux (3%, 10%, and 30% of the Crab) have also been considered.

We show that the detection probability not only depends on the total amount of observing time, but also on the length of the observing windows. Strategies that split the observing time into many small observations along the light curve yield a higher detection probability compared to ones that concentrate the observations in few large windows. This is a consequence of positive fluctuations in the sources' flux which are more likely to be observed when many observing windows are employed.

The best observing strategy for the fast detection of a source is found by balancing the number of telescopes composing each sub-array and the minimum amount of time each sub-array need to detect the source. In this case the use of the small sub-array and 10 minute observing windows allows
for the fastest detection of the source. For sources with mean fluxes of 3%, 10% and 30% of the Crab, the simultaneous use of five small sub-arrays represent the best option for monitoring, as it allows for the maximum number of sources to be monitored within one night. Many aspects that influence observations with Cherenkov telescopes (visibility of the sources, moonlight, weather conditions) have not been taken into account in this work. As a consequence, these results should be regarded more as recommendations of a good way to utilize CTA’s full potential, rather than a strict prescription on how to use it.

The influence of light curves’ properties on the choice of the best strategies is studied by varying the light curve parameters one at a time around the chosen reference values. It appears that light curve’s variance and PSD index have a strong influence on the detectability of the sources and on the selection of the best strategy. This suggests that, upon preparing to observe a variable source, the properties of its light curve must be taken into account to prepare the most efficient observing plan. More over, the choice of a particular observing strategy favor or disfavor the detection of sources, depending on the characteristics of their light curves. This is something that needs to be taken into account when conducting extragalactic sky surveys, when the aim is to detect an unbiased sample of variable sources. The impact of the variability characteristics on the observing strategies is only briefly tested here. Our preliminary study can be the starting point of a more detailed and exhaustive study.

During this thesis, great effort has been made to create a tool for observing strategy optimization that could be easily extended and upgraded. This tool has been developed in order to allow an easy implementation of different observing strategies and observing goals. The work presented here is planned to be published in a technical paper.
Bibliography


[22] https://wwwmagic.mpp.mpg.de/.


[48] https://znwiki3.ifh.de/CTA/Eventdisplay


