Surface Tension and Soap Bubbles

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Surface tension can be explained in terms of molecular forces [1]. In Fig. 1 a plot of the intermolecular force as a function of distance is given. A molecule (blue circle) is located at the origin of a reference frame and a second one at a distance $d_0$ along the $x$ axis. Let us suppose the latter can move, varying the distance from the former and experiencing different forces. For $d = d_0$ no force is present. For $d < d_0$ there is a repulsive force that prevents the couple of molecules from collapsing. For $d > d_0$ the molecules are attracted each other up to a distance $d_0 = a$; the interaction becomes negligible for $x > a$.

When two phases (liquid and gas for example) are in contact, there is a molecule-wide transition region in which density rapidly changes. A pure liquid in contact with its vapour exhibits a more gradual and smooth transition, extending up to five or six molecular diameters (Fig. 2).

Molecules within a fluid move continuously with a chaotic motion (also known as Brownian motion): consequently every molecule of the fluid (liquid or gas) interacts with the neighbouring molecules in all directions. Averaged over a long time scale (if compared with the typical time of the chaotic molecular displacement) the resulting force upon whatever a molecule is zero.

![Fig. 1 - Behaviour of the force between two molecules as a function of distance. The distance $d_0$ corresponds approximately to the molecular diameter. For water, $d_0 \approx 0.3$ nm, but this value strongly depends on the molecular structure. The distance $a$, where the interaction becomes negligible, corresponds to about 100 $d_0$.](image)

On the contrary, at the boundary between two phases, the interaction among neighbouring molecules has not the same intensity in all directions. Due to closer molecular packing, attractive and repulsive forces have a greater importance in the liquid phase than in the gaseous phase. For a molecule near the surface of the liquid the force is directed toward the liquid; in fact much more molecules are present on that side. Therefore, the surface of the liquid can be compared to a stretched membrane under tension and the presence of this membrane can be associated to a storage of energy: a steel needle can be made to float on the surface of water even though steel is more dense than water. For a gas the intensity of this cohesion force is much smaller. This microscopic description is helpful to understand in detail the situation at the liquid-gas surface, also named interphase or interface.

From the macroscopic point of view, in order to produce a new surface, some work $w$ must be done on the system to move molecules from the bulk to the surface:

$$w = \tau \Delta S$$

(1)

where $\tau$ is called Surface Tension or Free Surface Energy and $\Delta S$ is the area of the increased surface. For a given volume, the sphere has the surface with the minimum area: for this reason both liquid droplets and gas bubbles are spherical.
In a droplet the surface contraction must be balanced by a pressure increase: the work done by the pressure forces, when the volume changes by an amount $\Delta V$, is $w = p \Delta V$. For a sphere, $\Delta S$ and $\Delta V$ can be easily calculated: $\Delta S = 8\pi r \Delta r$ and $\Delta V = 4\pi r^2 \Delta r$. As a consequence, by using also Eq. 1, the Laplace’s equation can be immediately obtained:

$$\Delta p = \frac{2\tau}{r}$$

(2)

It gives the difference $\Delta p$ of the liquid pressure with respect to the gas, due to the curvature of the surface. In case of a hollow sphere (soap bubble) the Laplace’s equation becomes $\Delta p = \frac{4\tau}{r}$, because in this situation the total surface is $8\pi r^2$ (inner and outer surface of the sphere).

Whether or not a liquid wets a solid surface is determined by the relative strength of the cohesive forces among the molecules of the liquid compared to the adhesive forces among the molecules of the liquid and those of the container\(^1\). Water wets glass because the water molecules are more strongly attracted by the glass molecules than they are by other water molecules. Water rises in a capillary, but the general behaviour of a liquid in a capillary depends on the relative strengths of adhesive and cohesive forces.

Water droplets are more or less spherical. In case of small droplets deposited on a surface, gravity has a little effect and the droplet is approximately spherical. With a greater amount of liquid a distortion of the droplet profile can be observed.

Surface tension is always very attractive in physics courses. Students are fascinated even by simple demonstrations: for instance soap films deposited upon thin frames with different geometrical shapes are really eye-catching and represent a simple way to teach the surfaces of minimal area for given boundary conditions. The most common techniques used to measure the water’s surface tension are the drop-weight method and the ring method.

References


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\(^1\) The word *cohesion* applies to forces acting among molecules of the same type, while *adhesion* is used for molecules of different types.